Differentiate and simplify where appropriate:

1. (5pts) $\dfrac{d}{dx} \, 13^{x^2 - 5x + 7} =$

2. (8pts) $\dfrac{d}{dt} \, \ln \sqrt[5]{\dfrac{t^2}{3t + 2}} =$

3. (8pts) $\dfrac{d}{du} \left(u \arctan u - \dfrac{1}{2} \ln(1 + u^2) \right) =$

4. (7pts) (note this is not a product, let $x > 0$) $\dfrac{d}{dx} \arcsin \left(\sqrt{1 - x^2} \right) =$

5. (8pts) Draw the graph of a function that is continuous and differentiable on $[2, 9]$ which satisfies:

$f'(x) > 0$ on $(2, 4)$
$f'(x) < 0$ on $(4, 7)$
$f'(x) > 0$ on $(7, 9)$
$f(4) = 5, \; f(7) = 3$

6. (12pts) Use Rolle's Theorem to show that the equation $x^3 + e^x = 0$ has at most one solution.

7. (14pts) Let $f(x) = \cos^3 x - \sin^3 x$. Find the absolute minimum and maximum values of f on the interval $[0, 2\pi]$.

8. (10pts) Use logarithmic differentiation to find the derivative of $y = (\cos x)^{\cos x}$.

9. (12pts) Let $f(x) = x^2 - 8x + 15$, $x \leq 4$, and let g be the inverse of f. Use the theorem on derivatives of inverses to find $g'(3)$.

10. (16pts) The angle of elevation is the angle between the ground and the line joining an object with the observer. An outside elevator that is descending at rate 2 meters per second is watched by an observer located on the ground 300 meters from the foot of the building. At what rate is the angle of elevation changing when the elevator is 60 meters above ground?

Bonus. (10pts) Let $f(x)$ be function defined on $\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$ which satisfies: $f'(x) = \cos^2 x$ and $f\left(\frac{\pi}{6}\right) = 0$. Use the Mean Value Theorem to show that $\frac{1}{4}x - \frac{\pi}{24} \leq f(x) \leq \frac{3}{4}x - \frac{\pi}{8}$ on the interval $\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$.