

Calculus 1 — Exam 2  
MAT 250, Spring 2012 — D. Ivanšić

Name: \_\_\_\_\_  
*Show all your work!*

Differentiate and simplify where appropriate:

1. (6pts)  $\frac{d}{dx} \left( 2x^7 - \frac{5}{x^3} + \sqrt[4]{x^7} + e^2 \right) =$

2. (6pts)  $\frac{d}{dt} (t^2 + yt)e^t =$

3. (8pts)  $\frac{d}{dx} \frac{3x - 1}{x^3 - 5x^2 + 17} =$

4. (9pts)  $\frac{d}{dw} \frac{w + \sqrt[4]{w}}{w - \sqrt[4]{w}} =$

5. (6pts) Let  $h(x) = \frac{f(x) + g(x)}{f(x)g(x)}$ . Find the general expression for  $h'(x)$  and simplify.

Find the following limits algebraically.

6. (5pts)  $\lim_{x \rightarrow 3} \frac{x^2 + x - 12}{x^2 - 10x + 21} =$

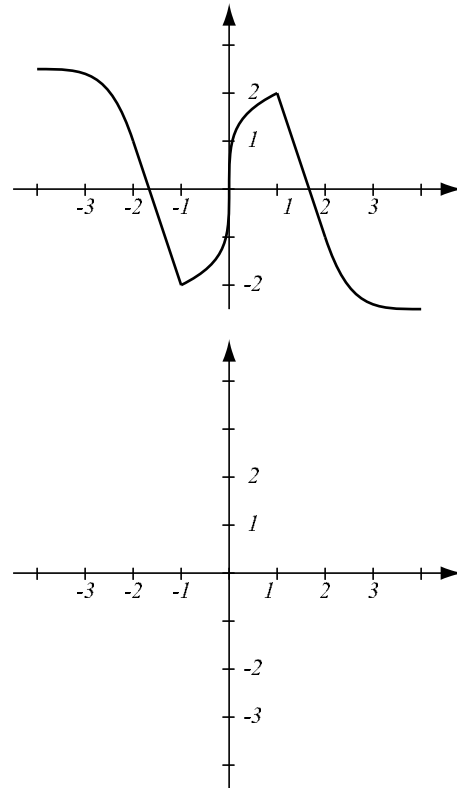
7. (7pts)  $\lim_{x \rightarrow 25} \frac{5 - \sqrt{x}}{25 - x} =$

8. (7pts)  $\lim_{x \rightarrow 0} \frac{\sin(3x)}{x^2 - x} =$

9. (10pts) Find  $\lim_{x \rightarrow 0^+} x^3 \left( 4 + \sin^2 \left( \frac{1}{x} \right) \right)$ . Use the theorem that rhymes with what unkind children do to their peers.

10. (12pts) The graph of the function  $f(x)$  is shown at right.

- Find the points where  $f'(a)$  does not exist.
- Use the graph of  $f(x)$  to draw an accurate graph of  $f'(x)$ .
- Is  $f(x)$  odd or even? How about  $f'(x)$ ?



11. (16pts) Let  $f(x) = \frac{x}{x+1}$ .

- Use the limit definition of the derivative to find the derivative of the function.
- Check your answer by taking the derivative of  $f$  using rules.
- Write the equation of the tangent line to the curve  $y = f(x)$  at point  $(1, \frac{1}{2})$ .

**12.** (8pts) Consider the limit below. It represents a derivative  $f'(a)$ .

a) Find  $f$  and  $a$ .

b) Use the information above and differentiation formulas to find the limit.

$$\lim_{x \rightarrow 32} \frac{\sqrt[5]{x} - 2}{x - 32}$$

**Bonus.** (10pts) We have indicated how to prove  $(x^n)' = nx^{n-1}$  for  $n \geq 0$ . Show that the formula works for integers  $n < 0$  as follows: set  $n = -k$ , and develop the rule for the derivative of  $x^{-k}$  with the help of the quotient rule and the rule for positive exponents.

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<sup>0</sup>Total points: 200