

Calculus 1 — Exam 1
MAT 250, Spring 2012 — D. Ivanšić

Name: _____
Show all your work!

1. (16pts) Use the graph of the function to answer the following. Justify your answer if a limit does not exist.

$$\lim_{x \rightarrow -1^-} f(x) =$$

$$\lim_{x \rightarrow -1^+} f(x) =$$

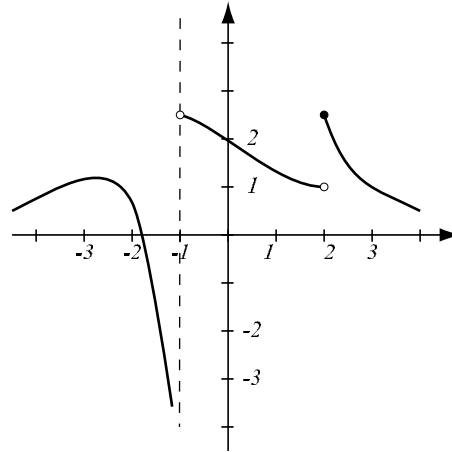
$$\lim_{x \rightarrow 2^-} f(x) =$$

$$\lim_{x \rightarrow 2^+} f(x) =$$

$$\lim_{x \rightarrow 2} f(x) =$$

$$f(2) =$$

List points where f is not continuous and justify why it is not continuous at those points.



2. (4pts) Find the following limit algebraically (no need to justify, other than showing the computation).

$$\lim_{x \rightarrow 2} \cos(x^2 + 5x - 14) =$$

3. (8pts) Let $\lim_{x \rightarrow 5} f(x) = -7$ and $\lim_{x \rightarrow 5} g(x) = 3$. Use limit laws to find the limit below and show each step.

$$\lim_{x \rightarrow 5} \frac{x - 3g(x) - 2}{f(x) \cdot g(x) + x} =$$

4. (14pts) Let $f(x) = \frac{3x^2 + x - 1}{x - 3}$.

- a) Find the domain of f .
- b) Explain, using continuity laws, why the function is continuous on its domain.
- c) At points of discontinuity, state the type of discontinuity (jump, infinite, removable) and justify.

5. (16pts) The temperature in degrees Celsius of a rotisserie chicken left to cool is given by $f(t) = 0.15t^2 - 6t + 100$, where t is in minutes.

- a) Find the average rates of change of temperature of the chicken over six short intervals of time, three of them beginning with 4, and three ending with 4. Show the table of values. What are the units?
- b) Use the information in a) to find the instantaneous rate of cooling of the chicken at $t = 4$. What are the units?

Drool here as needed

6. (14pts) Find an interval of length at most 0.01 that contains the solution of the equation $\ln x = 7 - x$. Use the Intermediate Value Theorem to justify why your interval contains the solution.

7. (16pts) Consider the limit $\lim_{x \rightarrow 0} \frac{\cos x - 1}{x^2}$.

- Can you use a limit law to find the limit? Why or why not?
- Fill out the table of values below. What do you think is the limit to six decimals accuracy?
- Explain the unusual values at the bottom of the table.

x	$\frac{\cos x - 1}{x^2}$
0.1	
0.01	
0.001	
10^{-4}	
10^{-5}	
10^{-6}	
10^{-7}	

8. (12pts) Draw the graph of a function, defined on the interval $(-3, 4)$ that exhibits the following features:

$$\lim_{x \rightarrow -3^+} f(x) = \infty$$

$$\lim_{x \rightarrow 4^-} f(x) = 3$$

$$\lim_{x \rightarrow 2^-} f(x) = -1$$

$$\lim_{x \rightarrow 2^+} f(x) = 5$$

$f(x)$ is right-continuous at $x = 2$

Bonus. (10pts) Below is the graph of the position of a car t minutes after noon. Arrange the numbers a , b , c and d in increasing order (some may be equal) and justify.

a = slope of secant line through points A and B

b = instantaneous velocity at time $t = 3$

c = slope of tangent line at point B .

d = average velocity over the interval $[3, 8]$.

