Calculus 1 — Exam 6
MAT 250, Spring 2013 — D. Ivanšić

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Show all your work!

Find the following antiderivatives.

1. (3pts) $\int \frac{1}{\sqrt{x}}\,dx =$ $\int x^{-\frac{1}{2}}dx = \frac{x^{\frac{1}{2}}}{\frac{1}{2}} = 2x^{\frac{1}{2}} = 2\sqrt{x} + C$

2. (3pts) $\int \frac{1}{\sqrt{1-x^2}}\,dx =$ $\arcsin x + C$

3. (4pts) $\int \sec(4\theta)\tan(4\theta)\,d\theta =$ $\frac{\sec(4\theta)}{4} + C$

4. (7pts) $\int t^2(t - \sqrt[4]{t})\,dt =$ $\int t^3 - t^{\frac{9}{4}}\,dt = \frac{t^4}{4} - \frac{t^{\frac{13}{4}}}{\frac{13}{4}} = \frac{t^4}{4} - \frac{4}{13}t^{\frac{13}{4}} + C$

5. (8pts) Find $f(x)$ if $f'(x) = x^{\frac{2}{3}} + \frac{7}{x}$ and $f(1) = 6$.

$f'(x) = x^{\frac{2}{3}} + \frac{7}{x}$

$f(x) = \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + 7\ln|x| + C$

$= \frac{3}{5}x^{\frac{5}{3}} + 7\ln|x| + C$

$6 = f(1) = \frac{3}{5} + C$

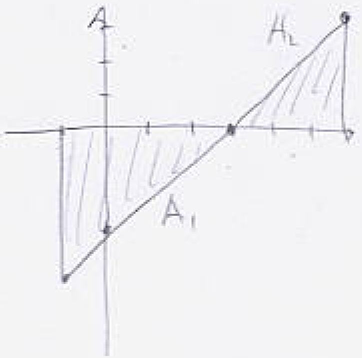
$C = 6 - \frac{3}{5} = \frac{27}{5}$

$f(x) = x^{\frac{2}{3}} + 7\ln|x| + \frac{27}{5}$

6. (6pts) Write using sigma notation:

$\frac{1}{1} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{5}} + \cdots + \frac{1}{\sqrt{101}} =$ $\sum_{i=0}^{50} \frac{1}{\sqrt{2i+1}} = \sum_{k=1}^{51} \frac{1}{\sqrt{2k-1}}$

7. (16pts) Find $\int_{-1}^{6} x - 3\,dx$ in two ways (they'd better give you the same answer!):
a) Using the "area" interpretation of the integral. Draw a picture.
b) Using the Evaluation Theorem.



a) $\int_{-1}^{6} x-3\,dx = -A_1 + A_2 = -\frac{1}{2}4\cdot4 + \frac{1}{2}3\cdot3 = -8 + \frac{9}{2} = -\frac{7}{2}$

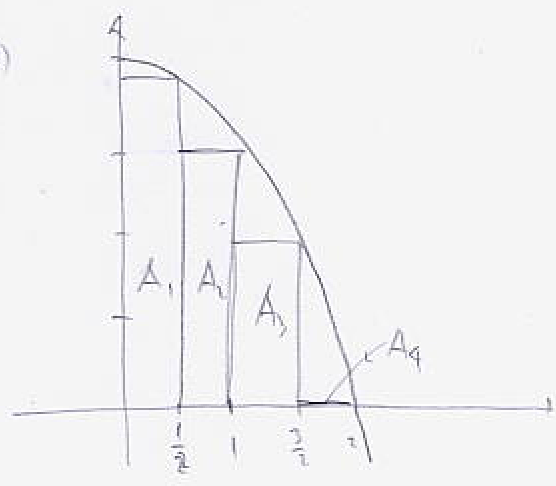
b) $\int_{-1}^{6} x-3\,dx = \left(\frac{x^2}{2} - 3x\right)\Big|_{-1}^{6} = \left(\frac{x^2}{2}\right)\Big|_{-1}^{6} - 3x\Big|_{-1}^{6}$

$= \frac{1}{2}\left(6^2 - (-1)^2\right) - 3(6-(-1)) = \frac{35}{2} - 21 = \frac{35-42}{2} = -\frac{7}{2}$

same!

8. (16pts) The function $f(x) = 4 - x^2$, $0 \le x \le 2$ is given.
a) Write down the expression R_4 that estimates the area under this curve using four approximating rectangles and right endpoints. Then evaluate the expresssion.
b) Illustrate with a diagram, where appropriate rectangles are clearly visible. What does R_4 represent? Does it over- or underestimate the area under the curve?

b)



a) $R_4 = \frac{1}{2}\left(f(\tfrac{1}{2}) + f(1) + f(\tfrac{3}{2}) + f(2)\right)$

$= \frac{1}{2}\left(4 - \frac{1}{4} + 4 - 1 + 4 - \frac{9}{4} + 0\right)$

$= \frac{1}{2}\left(11 - \frac{5}{2}\right) = \frac{17}{4}$

$\Delta y = \frac{1}{2}$

R_4 = sum of areas of rectangles $= A_1 + A_2 + A_3 + A_4$

It underestimates the actual area.

Use the substitution rule in the following integrals:

9. (9pts) $\int (x-4)\cos(x^2-8x+4)\,dx =$ $\begin{bmatrix} u = x^2-8x+4 \\ du = 2x-8\,dx \\ \frac{1}{2}du = x-4\,dx \end{bmatrix} = \int \cos u \frac{1}{2}\,du$

$= \frac{1}{2}\sin u = \frac{1}{2}\sin(x^2-8x+4)+C$

10. (10pts) $\int_0^1 \frac{e^x+1}{(e^x+x)^2}\,dx =$ $\begin{bmatrix} u = e^x+x & x=1,\ u=e+1 \\ du = e^x+1\,dx & x=0,\ u=1 \end{bmatrix} = \int_1^{e+1} \frac{1}{u^2}\,du$ u^{-2}

$= -u^{-1}\Big|_1^{e+1} = -\frac{1}{u}\Big|_1^{e+1} = -\left(\frac{1}{e+1}-1\right) = 1-\frac{1}{e+1}$

11. (10pts) The rate at which a deer population is growing is $2+\frac{t}{4}$ deer per day.
a) Use the Net Change Theorem to find how much the population has grown in 8 days.
b) If there were initially 107 deer, how many were there after 8 days?

a) $\Delta P = \int_0^8 P'(t)\,dt = \int_0^8 2+\frac{t}{4}\,dt = \left(2t+\frac{t^2}{8}\right)\Big|_0^8 = \left(16+\frac{64}{8}\right)-0 = 24$ deer

In 8 days, population increases by 24 deer.

b) $107+24=131$ deer

12. (8pts) Show that $2 \le \int_{-1}^{1} e^{x^2}\,dx \le 2e$ **without** evaluating the integral.

$-1 \le x \le 1$

so $0 \le x^2 \le 1$

$e^0 \le e^{x^2} \le e^1$

$1 \le e^{x^2} \le e$

$1\cdot(1-(-1)) \le \int_{-1}^{1} e^{x^2}dx \le e(1-(-1))$

$2 \le \int_{-1}^{1} e^{x^2}dx \le 2e$

Bonus. (10pts) A car is traveling at velocity 3 meters per second when it starts accelerating at constant acceleration. If it has traveled 108 meters during the 6 seconds that it accelerated, what is its acceleration?

Let A be the constant acceleration of the car.

$a(t) = A, \quad v(0) = 3, \quad s(0) = 0$

$v(t) = At + C$

$3 = v(0) = 0 + C$

so $C = 3$

$v(t) = At + 3$

$s(t) = A\frac{t^2}{2} + 3t + D$

$0 = s(0) = 0 + 0 + D$

so $D = 0$

$s(t) = \frac{A}{2}t^2 + 3t$

$108 = s(6) = \frac{A}{2}\cdot 6^2 + 3\cdot 6$

$108 = 18A + 18 \quad |-18$

$90 = 18A$

$A = \frac{90}{18} = 5\ m/s^2$