Calculus 1 — Exam 1
MAT 250, Spring 2013 — D. Ivanšić

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Show all your work!

1. (16pts) Use the graph of the function to answer the following. Justify your answer if a limit does not exist.

$\lim_{x\to -2^-} f(x) = -\infty$

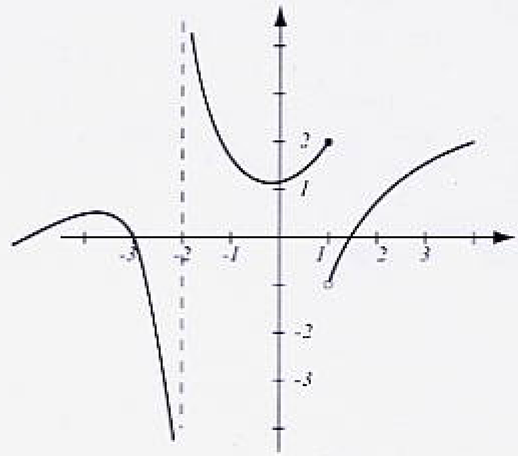
$\lim_{x\to -2^+} f(x) = \infty$

$\lim_{x\to -2} f(x) =$ d.n.e (one-sided limits not equal)

$\lim_{x\to 1^+} f(x) = -1$

$\lim_{x\to 1} f(x) =$ d.n.e (one-sided limits are not equal)

$f(1) = 2$



List points where f is not continuous and justify why it is not continuous at those points.

f is not continuous at:

$x=-2$, because $f(-2)$ is not defined

$x=1$, because $\lim_{x\to 1} f(x)$ d.n.e,

2. (8pts) Let $\lim_{x\to 2} f(x) = 3$ and $\lim_{x\to 2} g(x) = -1$. Use limit laws to find the limit below and show each step.

$$\lim_{x\to 2}\sqrt{\frac{xf(x)-4}{x^3+g(x)}} = \sqrt{\lim_{x\to 2}\frac{xf(x)-4}{x^3+g(x)}} = \sqrt{\frac{\lim_{x\to 2}(xf(x)-4)}{\lim_{x\to 2}(x^3+g(x))}} = \sqrt{\frac{\lim_{x\to 2} x \lim_{x\to 2} f(x) - \lim_{x\to 2} 4}{(\lim_{x\to 2} x)^3 + \lim_{x\to 2} g(x)}}$$

$$= \sqrt{\frac{2\cdot 3-4}{2^3+(-1)}} = \sqrt{\frac{2}{7}}$$

3. (10pts) Find $\lim_{x\to 0}\frac{x^2}{4+\sin\left(\frac{1}{x}+3\right)}$. Use the theorem that rhymes with an exclamation conveying surprise and derision.

$$-1 \le \sin\left(\tfrac{1}{x}+3\right) \le 1$$

$$3 \le 4+\sin\left(\tfrac{1}{x}+3\right) \le 5$$

$$\frac{1}{3} \ge \frac{1}{4+\sin\left(\frac{1}{x}+3\right)} \ge \frac{1}{5}$$

$$\frac{x^2}{3} \ge \frac{x^2}{4+\sin\left(\frac{1}{x}+3\right)} \ge \frac{x^2}{5}$$

$\left.\begin{array}{l}\lim_{x\to 0}\frac{x^2}{3}=0 \\ \lim_{x\to 0}\frac{x^2}{5}=0\end{array}\right\}$ Since these are equal, by the squeeze theorem we have $\lim_{x\to 0}\frac{x^2}{4+\sin\left(\frac{1}{x}+3\right)}=0$

Find the following limits algebraically. Do not use the calculator.

4. (5pts) $\lim\limits_{x\to 4} \dfrac{x^2-16}{x^2-5x+4} = \lim\limits_{x\to 4} \dfrac{(x-4)(x+4)}{(x-4)(x-1)} = \lim\limits_{x\to 4} \dfrac{x+4}{x-1} = \dfrac{8}{3}$

5. (7pts) $\lim\limits_{x\to 13} \dfrac{\sqrt{x+3}-4}{x-13} = \lim\limits_{x\to 13} \dfrac{\sqrt{x+3}-4}{x-13} \cdot \dfrac{\sqrt{x+3}+4}{\sqrt{x+3}+4} = \lim\limits_{x\to 13} \dfrac{\overbrace{x+3-16}^{x-13}}{(\cancel{x-13})(\sqrt{x+3}+4)}$

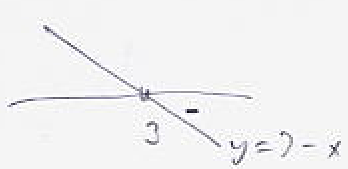
$= \lim\limits_{x\to 13} \dfrac{1}{\sqrt{x+3}+4} = \dfrac{1}{8}$

6. (6pts) $\lim\limits_{x\to 0} \dfrac{\tan x}{x} = \lim\limits_{x\to 0} \dfrac{\frac{\sin x}{\cos x}}{x} = \lim\limits_{x\to 0} \dfrac{\sin x}{\cos x} \cdot \dfrac{1}{x} = \lim\limits_{x\to 0} \overbrace{\dfrac{\sin x}{x}}^{\to 1} \cdot \underbrace{\dfrac{1}{\cos x}}_{\to 1} = 1$

7. (7pts) $\lim\limits_{x\to\infty} \dfrac{5x^2-3x+1}{4x^3-4x^2+7} = \lim\limits_{x\to\infty} \dfrac{\cancel{x^2}\left(5-\frac{3}{x}+\frac{1}{x^2}\right)}{\underset{x}{\cancel{x^3}}\left(4-\frac{4}{x}+\frac{7}{x^3}\right)} = 0 \cdot \dfrac{5-0+0}{4-0+0} = 0 \cdot \dfrac{5}{4} = 0$

$\frac{1}{x} \to 0$

8. (5pts) $\lim\limits_{x\to 3^+} \dfrac{2x+1}{3-x} = \dfrac{7}{0^-} = -\infty$



When $x > 3$, $3-x < 0$

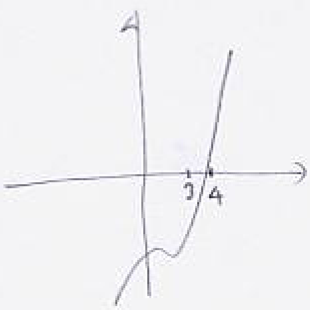
9. (14pts) Use your calculator to find an interval of length at most 0.01 that contains the solution of the equation $x^3 - 4x^2 + 3x = 8$. Use the Intermediate Value Theorem to justify why your interval contains the solution.

Let $f(x) = x^3 - 4x^2 + 3x - 8$. It is continuous everywhere (it is a polynomial)

By examining the graph we see to look for a zero near $x = 4$

$f(3.76) = -0.113024$
$f(3.77) = 0.041033$

Since 0 is between $f(3.76)$ and $f(3.77)$, by IVT there is a c in $(3.76, 3.77)$ such that $f(c) = 0$.



10. (10pts) Consider the limit $\lim_{x \to 1} \dfrac{2^x - 2}{x - 1}$. Use your calculator to estimate this limit with accuracy 4 decimal points. Write a table of values that will justify your answer.

x	$\dfrac{2^x - 2}{x - 1}$
1,001	1,38677..
1,0001	1.38634
1,00001	1,386299..
1,000001	1.3862944
1,0000001	1,386294..
0,999	1.38581..
0,9999	1.38624..
0.99999	1,38628..
0.999999	1.38629..
0.9999999	1,38629..

both appear to stabilize around 1,3862

so that is probably the limit with accuracy 4 decimals

11. (12pts) Draw the graph of a function, defined on the interval $(-3, 4)$ that exhibits the following features:

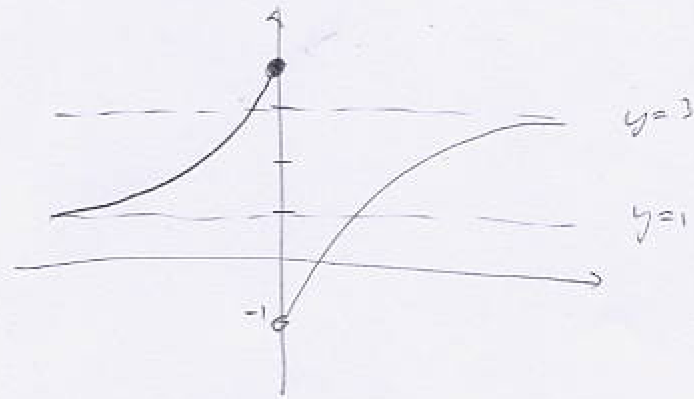
$\lim_{x\to\infty} f(x) = 3$

$\lim_{x\to-\infty} f(x) = 1$

$\lim_{x\to 0^-} f(x) = 4$

$\lim_{x\to 0^+} f(x) = -1$

$f(x)$ is left-continuous at $x = 0$



Bonus. (10pts) Show that $\frac{0}{0}$ is an indeterminate form. That is, come up with three pairs of functions $f(x)$, $g(x)$ such that $\lim_{x\to a} f(x) = 0$ and $\lim_{x\to a} g(x) = 0$ in every case, yet $\lim_{x\to a} \frac{f(x)}{g(x)}$ is different for the three cases. (Think of simple functions.)

Examples

$f(x)$	$g(x)$	$\lim_{x\to 0} \frac{f(x)}{g(x)}$
x	x^2	$\lim_{x\to 0} \frac{1}{x}$ d.n.e. (it's ∞ if $x\to 0+$, $-\infty$ if $x\to 0-$)
x^2	x	$\lim_{x\to 0} x = 0$
x	x	$\lim_{x\to 0} 1 = 1$

get different limits