## Mathematical Concepts - Joysheet 1 MAT 117, Spring 2013 - D. Ivanšić

Show all your work!

Use your calculator to compute each expression to 6 significant digits accuracy or six decimal places, whichever is more accurate. Write down the sequence of keys you entered in order to compute each expression. Do not round numbers in mid-computation.

1. $(5 \mathrm{pts}) \sqrt[5]{23}=$
2. (9pts) $1575\left(1+\frac{0.05}{4}\right)^{20}=$
3. $(7 \mathrm{pts}) 13(\sqrt[5]{4.25}-1)=$
4. $(6 \mathrm{pts}) \frac{\log 0.336}{\log 6.63}=$
5. $(9 \mathrm{pts}) \frac{\log 7.32}{4 \log 1.012}=$
6. $(12 \mathrm{pts}) \frac{\left(1+\frac{0.0275}{4}\right)^{28}-1}{\frac{0.0275}{4}}=$
7. $(12 \mathrm{pts}) \frac{1-\left(1+\frac{0.03875}{12}\right)^{-48}}{\frac{0.03875}{12}}=$

## Mathematical Concepts - Joysheet 2 <br> MAT 117, Spring 2013 - D. Ivanšić

1. (10pts) a) What percent of 7 is 4 ?

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b) $13 \%$ of what number is 20 ?
2. (6pts) You bought a pair of sunglasses for $\$ 139$. If sales tax is $6.25 \%$ (like in some counties in Illinois), what is the total cost?
3. (12pts) Jack and Jill, a married couple with one child, filed a tax return in 2008. Their total income was $\$ 76,300$, they deposited $\$ 6000$ into a retirement account, paid $\$ 4,500$ in mortgage interest and $\$ 2,400$ in property taxes, and donated $\$ 300$ to charity. Use the table on page 448 of our book to first determine Jack and Jill's taxable income (don't forget the exemption) and then find the tax on this income.
4. (12pts) In 2006, rent for a 1-bedroom apartment in a building was $\$ 600$. After three years, due to high demand, it climbed $35 \%$. Then, over the next three years, demand decreased significantly, so rent fell $25 \%$. What is the rent after the six-year period? Did it increase or decrease compared to rent in 2006 ?
5. (10pts) How much money should you deposit in a simple-interest account bearing $2.53 \%$ if you would like to have $\$ 3000$ in eighteen months? How much of the final $\$ 3000$ is from interest?
6. (10pts) You can deposit $\$ 1,000$ into an account bearing $4.75 \%$ simple interest. How long will it take until you have $\$ 1,300$ in the account?

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Mathematical Concepts - Joysheet 3
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Name:
Show all your work!

1. (8pts) Sabrina would like to have $\$ 5000$ for a decent used car. How much should she deposit now in an account bearing $2.66 \%$, compounded monthly, in order to have the desired amount in three and a quarter years? How much of the $\$ 5000$ came from interest?
2. (6pts) Bank of Hind Quarters is offering a $2.38 \%$ interest rate on an account that is compounded daily, while Rear End Bank has an account at $2.39 \%$, compounded quarterly. Which account is the better deal?
3. (10pts) An investment that you made quadrupled in value in 5 years. Assuming annual compounding, at what annual rate did this investment grow?
4. (10pts) To save for a trip to Mt. Everest in six years (approximate cost $\$ 50,000$ ), you make monthly deposits into an account bearing $6.5 \%$, compounded monthly.
a) How much should you deposit every month to reach your goal?
b) How much would you earn in interest over the six years?
5. (14pts) At the time of little Ruby's birth, her parents decided to save some money for her college. They only started to have extra money when Ruby was 4, when they started depositing $\$ 300$ every month into an account bearing $6.4 \%$ interest, compounded monthly. When Ruby was 14, however, new financial hardship forced the parents to stop their contributions, but they left the money in the account.
a) How much is in the account when Ruby is 18 ?
b) How much of it was from deposits, and how much from interest?
6. (12pts) Miguel would like to save $\$ 10,000$ to add a jacuzzi to his home. If he can set aside $\$ 350$ every month into an account bearing $5.41 \%$, compounded monthly, how long will it take him to save the desired amount?

> Mathematical Concepts - Joysheet 4 MAT 117, Spring 2013 - D. Ivanšić

This is an exercise in computing the payment on a hypothetical loan and comparing it with the numbers that financial services websites give you. Do the following:

1. (4pts) Decide on an amount and purpose for a hypothetical loan (e.g. buying a car, house, starting a business, etc.) Choose over how many years it should be repaid. Standard choices for each category are suggested: $15,20,30$ years for a home, $3,4,5$ years for a car, etc.
2. (14pts) Find a financial services website (bank, mortgage originator) that gives you interest rate quotes for the kind of loan that you chose in problem 1 and computes the monthly payment based on a loan amount. Use their computation to find the monthly payment on your hypothetical loan. Don't use a website with a "generic" calculator such as bankrate.com, rather, find one that offers actual loans with current interest rates. Print out the webpage, showing loan amount, term, interest rate and payment and attach it to this one. Try to keep it to just one sheet.
(Attachment)
3. (12pts) Using our loan formula from 8.5, compute (write the computation here) the monthly payment on your hypothetical loan. Use the interest rate that you found on the website. The frequency of compounding is typically monthly. Does your number agree with the information on the website you found?
4. (14pts) Find the balance of the hypothetical loan after two thirds of all payments have been made.
5. (16pts) Write an amortization schedule for the four payments after two thirds of all payments have been made. (For example, if it's a 60 -month loan, consider payments 41,42 , 43 and 44.)

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1. (15pts) Do this part on your own. Roll two dice 50 times.
a) Record how many times you get each of the possible sums on the dice in the first row.
b) In the second row, enter the empirical probabilities for each sum based on your 50 rolls. Then compute the theoretical probabilities for each sum and enter them in the third row of the table. Enter these numbers as fractions.
c) Find the difference between the rows $P_{E}$ and $P_{T}$ and write it in decimal form rounded to 4 decimal places, ignoring any minus signs (that is what || stands for).

| Sum on roll | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Times occured |  |  |  |  |  |  |  |  |  |  |  |
| Empirical prob. $P_{E}$ |  |  |  |  |  |  |  |  |  |  |  |
| Theoretical prob. $P_{T}$ |  |  |  |  |  |  |  |  |  |  |  |
| $\left\|P_{E}-P_{T}\right\|$ |  |  |  |  |  |  |  |  |  |  |  |

2. (15pts) Do this part with 3 classmates. Write their names in the space provided. Each of you has to fill in the table independently, but the last three rows of this table should be the same for everyone in your group (check!).
a) Copy the "Times occured" line from above into row "You" and do the same for each of your classmates.
b) Sum by column and enter the sums in the row "Total times occured".
c) Write the empirical probability for each sum on the dice as a fraction. Keep in mind that your number of experiments is now larger.
d) Find $\left|P_{E}-P_{T}\right|$ and write it in decimal form rounded to 4 decimal places. Are the numbers smaller than in the table above?

| Sum on roll | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| You |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
| Total times occured |  |  |  |  |  |  |  |  |  |  |  |
| Empirical prob. $P_{E}$ |  |  |  |  |  |  |  |  |  |  |  |
| $\left\|P_{E}-P_{T}\right\|$ |  |  |  |  |  |  |  |  |  |  |  |

3. (10pts) A certain type of car is sold in seven colors, three trim levels, and you can choose exactly one of four different accessory packages to add to your car (or you may choose not to add an accessory package). If each of the features may be selected independently of the other, how many different car configurations can you buy?
4. (10pts) Suppose a bank card has nine digits, where the first one is always 5 , and the second one cannot be a zero. The last digit is a "check-digit", whose value is number of odd numbers among the remaining digits. (For example, if the first eight digits are 51312547, the ninth digit is 6 , since there are 6 odd digits in the group.) How many different bank cards can be issued?
5. (10pts) A coin is tossed, then a die is rolled, then a coin is tossed, and then a die is rolled.
a) How many different outcomes are there to this experiment?
b) How many different outcomes have 4 or 5 on the second roll of the die?

## Mathematical Concepts - Joysheet 6

 MAT 117, Spring 2013 - D. Ivanšić $\qquad$1. (10pts) In 2013, the College of Cardinals met to elect a pope. The table below breaks down the make-up of the College of Cardinals by age and region of origin.

If a current member of the college is selected,

| Age | Europe | Americas | Other |
| :---: | :---: | :---: | :---: |
| Under 70 | 26 | 17 | 11 |
| Over 70 | 34 | 16 | 11 | what is the probability that the cardinal

a) is from the Americas?
b) is over 70 ?
c) is over 70 and from the Americas?
d) is under 70 and not from Europe?
2. (20pts) Write the probabilities and odds against and in favor of the following events (show any work needed below):

| Event | probability | odds against | odds in favor |
| :--- | :--- | :--- | :--- |
| a) Rolling a 2 on a die |  |  |  |
| b) Drawing a 4 or 9 from a deck of cards |  |  |  |
| c) Drawing an ace or a red face card from a deck of cards |  |  |  |
| d) Getting numbers whose difference is 3 on a roll of two dice |  |  |  |
| e) Getting two heads in succession (anywhere) on three coin tosses |  |  |  |

3. (4pts) The odds in favor that a random college student will travel to a coastal location during spring break are 11-to-70.
a) What is the probability a student travels to a coastal location?
b) What is the probability a student doesn't travel to a coastal location?
4. (4pts) $74 \%$ of all gas station restrooms are reasonably clean.
a) What are the odds in favor of choosing a gas station with a clean restroom?
b) What are the odds against choosing a gas station with a clean restroom?
5. (12pts) A coin is tossed five times in succession. Write the number of outcomes of this experiment and then compute the probability that you got
a) exactly one head?
b) a head on the second toss or a tail on the fourth toss?
c) at least one of the outcomes is a tail?
6. (10pts) Among 31 LEGO sets in a child's possession, 15 contain a green piece, 12 contain a wheel piece and 21 contain a green piece or a wheel. What is the probability that a randomly chosen set
a) contains both a green piece and a wheel?
b) neither contains a green piece nor a wheel?

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Show all your work!

1. (14pts) Your schedule has classes $A$ and $B$ that meet on the same days. On any day, the probability that there is a pop quiz in class is 0.09 for class $A$ and 0.07 for class $B$, and these events are independent. What is the probability that
a) On a random day, class $A$ has a pop quiz and class $B$ doesn't?
b) On a random day, both classes have pop quizzes?
c) On two random days, at most three pop quizzes were given? Assume that in either class, whether quizzes are given on two days are independent events.
2. (14pts) Three cards are drawn from a standard deck without replacement. What is the probability that:
a) The second is a black face card, given that the first one was a black jack?
b) The first card is an ace and the second is an 8 or a 9 ?
c) All three are kings?
d) At least one is a face card?
3. (10pts) The table shows the make-up of a candy box with respect to whether the candies are milk or dark chocolate and what kind of filling they have (orange, raspberry, or none). What is the probability that a random candy:
a) has an orange filling?
b) is dark chocolate?
c) is milk chocolate, with raspberry filing?
d) has an orange filling, given it is dark chocolate?
e) is milk chocolate, given it has no filling?

| Type | Milk | Dark | Total |
| :---: | :---: | :---: | :---: |
| Orange | 7 | 3 |  |
| Raspberry | 10 | 4 |  |
| None | 12 | 8 |  |
| Total |  |  |  |

4. (8pts) A 25-year old can purchase a one-year life insurance policy for $\$ 10,000$ at a cost of $\$ 100$. Past history indicates that the probability of a person dying at age 25 is 0.002 .
a) Determine the company's expected gain per policy.
b) How much profit could a company expect if they insure 15,00025 -year olds?
5. (14pts) Fifty celebrities are divided into groups $A, B$ and $C$. A game of chance is played as follows: a player pays $\$ 500$ and collects $\$ 300$ if the first to announce a pregnancy within three months is a celebrity from group $A, \$ 700$ if they are from group $B, \$ 1000$ if they are from group $C$ and nothing if no celebrity announces a pregnancy. It is estimated that the chances a celebrity from a given group announces first are $46 \%$ for group $A, 28 \%$ for group $B$, and $10 \%$ for group $C$.
a) Find the expected value of this game.
b) If you play this game 20 times, how much do you expect to win or lose?
c) What is the fair price of this game?

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Name:
Show all your work!

Final answers should have accuracy to 6 decimal places. Show some work how medians and means are computed. Giving only the answer will bring you few points.

1. (8pts) The cemetery of a town has reached capacity, so city officials are considering expanding it into an area adjoining a residential neighborhood. The local newspaper would like to find out how much support there is for the idea and decides to survey the population. Answer whether each of the following methods will produce a good, bad or questionable random sample and comment why.
agood Surveying visitors to the cemetery.badiffy
bgood Surveying random people from the phone book whose last names beginbad with A.iffy
cSurveying random people from the town's voter lists.badiffy
dgood Surveying residents of the neighborhood adjoining the proposed expansion.badiffy
2. (22pts) A survey of 27 smartphones found the following prices without contract. Do the following:
a) Construct a grouped frequency distribution with first class 0-99.
b) Draw a histogram for the data
c) Enter a representative value for each interval.
d) Estimate the mean of the data based on the frequency distribution.
e) Find the actual mean and compare your answer to e).
$149,129,159,165,89,599,345,299,235109,459,325,550,449,380,249,199,333,155$, $175,429,359,425,445,525,259,399$

| Class | Frequency | Rep. value |
| :--- | :--- | :--- |
|  |  |  |
|  |  |  |
|  |  |  |

3. (10pts) A small business examines how many cars are parked in their parking lot over 15 days. The numbers are listed below.
a) Find the midrange of the data. $3,4,7,8,5,8,4,4,8,9,11,12,3,6,6$
b) Find the median of the data.
c) Find the mean of the data.
4. (20pts) Over a year, a small town police department tracks the number of weekly arrests. The numbers are shown below. Do the following:
a) Draw a histogram for the data.
b) Find the midrange of the data.
c) Find the median of the data.
d) Find the mean of the data.

| Weekly <br> arrests | Frequency <br> (weeks) |
| :---: | :---: |
| 0 | 3 |
| 1 | 7 |
| 2 | 6 |
| 3 | 10 |
| 4 | 14 |
| 5 | 8 |
| 6 | 4 |

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Show all your work!

Final answers should have accuracy to 6 decimal places (or 4 decimal places for table-derived answers). Show some work how the mean and standard deviation are computed. Giving only the answer will bring you few points.

1. (18pts) A restaurant chain is reviewing scores on the question "quality of food" on a comment card. The scores were numbers 1-5 ( 5 being best), and the number of cards with a given score on the question is listed below.
a) Find the range of the scores.
b) Find the mean of the scores.
c) Find the standard deviation of the scores.

| Score | Frequency <br> (cards) |
| :---: | :---: |
| 1 | 2 |
| 2 | 7 |
| 3 | 13 |
| 4 | 12 |
| 5 | 8 |

2. (10pts) The wait time at a grocery store checkout between the hours 4 PM and 6 PM on a Saturday has been found to have mean 4 minutes, with standard deviation 1.25 minutes. Use the 68-95-99.7 rule (draw a picture) to find the percentage of patrons that waited:
a) between 4 and 6.5 minutes
b) under 2.75 minutes
c) over 6.5 minutes
d) between 1.5 and 5.25 minutes.
3. (6pts) A set of data items is normally distributed with mean 13 and standard deviation 2.3. Find the data items that correspond to the $z$-scores given below.
a) $z=0$
b) $z=0.6$
c) $z=-1.7$
4. (4pts) Car companies were rated for owner satisfaction by many surveys. Company A scored 22 points on a survey with mean 18 and standard deviation 3, and company B scored 64 points on a survey with mean 50 and standard deviation 9 . Use $z$-scores to determine which company did better.
5. (22pts) The weight of cows at a certain farm is normally distributed with mean 825 lbs and standard deviation 56 lbs. Draw a picture showing which area you are computing as you answer:
a) What percentage of cows weigh less than 900 lbs ?
b) What percentage of cows weigh more than 800 lbs ?
c) What is the percentile of a cow weighing 875 lbs ? What does this mean?
d) What is the probability that a random cow weighs between 700 and 750 lbs ?
