Mathematical Concepts — Joysheet 7 MAT 117, Spring 2013 — D. Ivanšić

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1. (14pts) Your schedule has classes A and B that meet on the same days. On any day, the probability that there is a pop quiz in class is 0.09 for class A and 0.07 for class B, and these events are independent. What is the probability that

a) On a random day, class A has a pop quiz and class B doesn't?

b) On a random day, both classes have pop quizzes?

c) On two random days, at most three pop quizzes were given? Assume that in either class, whether quizzes are given on two days are independent events.

a) $P(\text{quiz in } A \text{ AND no quiz in } B) = P(\text{quiz in } A) \cdot P(\text{no quiz in } B) = 0.09 \cdot 0.93 = 0.0837$

b) $P(\text{quiz in } A \text{ AND quiz in } B) = P(\text{quiz in } A) \cdot P(\text{quiz in } B) = 0.09 \cdot 0.07 = 0.0063$

c) $P(\text{at most three quizzes on 2 days}) = 1 - P(\text{four quizzes in two days})$

$= 1 - P((\text{quiz in } A \text{ on day } 1) \text{ AND } (\text{quiz in } A \text{ on day } 2) \text{ AND } (\text{quiz in } B \text{ on day } 1)$
$\text{AND } (\text{quiz in } B \text{ on day } 2))$

$= 1 - 0.09 \cdot 0.09 \cdot 0.07 \cdot 0.07 = 1 - 0.00003969 = 0.99996$

2. (14pts) Three cards are drawn from a standard deck without replacement. What is the probability that:

a) The second is a black face card, given that the first one was a black jack?

b) The first card is an ace and the second is an 8 or a 9?

c) All three are kings?

d) At least one is a face card?

52 cards
12 face cards, 6 black face card,

a) $P(\text{2nd black face} \mid \text{1st black jack}) = \dfrac{5}{51}$

8 such cards

b) $P(\text{1st ace AND 2nd is 8 or 9}) = P(\text{1st ace}) \cdot P(\text{2nd 8 or 9} \mid \text{1st ace})$

$= \dfrac{\cancel{4}^{\,1}}{\cancel{52}_{\,13}} \cdot \dfrac{8}{51} = \dfrac{8}{663} = 0.0120664$

c) $P(\text{1st king AND 2nd king AND 3rd king})$

$= P(\text{1st king}) \cdot P(\text{2nd king} \mid \text{1st king}) \cdot P(\text{3rd king} \mid \text{1st king AND 2nd king})$

$= \dfrac{4}{52} \cdot \dfrac{3}{51} \cdot \dfrac{2}{50} = \dfrac{1}{13} \cdot \dfrac{1}{17} \cdot \dfrac{1}{25} = \dfrac{1}{5525} = 0.000180995$

$= 0.552941$

d) $P(\text{at least one is face}) = 1 - P(\text{none are face}) = 1 - P(\text{1st not face AND 2nd not face AND 3rd not face})$

$= 1 - P(\text{1st not face}) P(\text{2nd not face} \mid \text{1st not face}) \cdot P(\text{3rd not face} \mid \text{1st \& 2nd not face}) = 1 - \dfrac{40}{52} \cdot \dfrac{39}{51} \cdot \dfrac{38}{50} = 1 - \dfrac{38}{85} = \dfrac{47}{85}$

3. (10pts) The table shows the make-up of a candy box with respect to whether the candies are milk or dark chocolate and what kind of filling they have (orange, raspberry, or none). What is the probability that a random candy:
a) has an orange filling?
b) is dark chocolate?
c) is milk chocolate, with raspberry filing?
d) has an orange filling, given it is dark chocolate?
e) is milk chocolate, given it has no filling?

Type	Milk	Dark	Total
Orange	7	3	10
Raspberry	10	4	14
None	12	8	20
Total	29	15	44

a) $\frac{10}{44} = \frac{5}{22}$ 0.227273 b) $\frac{15}{44}$ 0.340909 c) $\frac{10}{44} = \frac{5}{22}$ 0.227273 d) $\frac{3}{15} = \frac{1}{5}$ 0.2 e) $\frac{12}{20} = \frac{3}{5}$ 0.6

4. (8pts) A 25-year old can purchase a one-year life insurance policy for \$10,000 at a cost of \$100. Past history indicates that the probability of a person dying at age 25 is 0.002.
a) Determine the company's expected gain per policy.
b) How much profit could a company expect if they insure 15,000 25-year olds?

a)

Outcomes	net effect to company	prob.
dies	$100 - 10000 = -9900$	0.002
lives	100	0.998

$E = 0.002 \cdot (-9900) + 0.998 \cdot 100 = -19.8 + 99.8 = 80$

\$80 expected gain per policy

b) $15000 \cdot 80 = 1,200,000$

5. (14pts) Fifty celebrities are divided into groups A, B and C. A game of chance is played as follows: a player pays \$500 and collects \$300 if the first to announce a pregnancy within three months is a celebrity from group A, \$700 if they are from group B, \$1000 if they are from group C and nothing if no celebrity announces a pregnancy. It is estimated that the chances a celebrity from a given group announces first are 46% for group A, 28% for group B, and 10% for group C.
a) Find the expected value of this game.
b) If you play this game 20 times, how much do you expect to win or lose?
c) What is the fair price of this game?

Outcomes	net win	Prob.
group A	$300 - 500 = -200$	0.46
group B	$700 - 500 = 200$	0.28
group C	$1000 - 500 = 500$	0.10
none	-500	0.16

$\uparrow 1 - (0.46 + 0.28 + 0.10)$

a) $E = 0.46 \cdot (-200) + 0.28 \cdot 200 + 0.10 \cdot 500 + 0.16(-500)$
$= -92 + 56 + 50 - 80 = -66$ expect to lose 66 per play

b) $20 \cdot (-66) = -1320$
expect to lose 1320

c) $-66 + 500 = 434$