

Mathematical Concepts — Joysheet 6
MAT 117, Spring 2013 — D. Ivanić

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 Show all your work!

1. (10pts) In 2013, the College of Cardinals met to elect a pope. The table below breaks down the make-up of the College of Cardinals by age and region of origin.

Age	Europe	Americas	Other
Under 70	26	17	11
Over 70	34	16	11

total: 115

If a current member of the college is selected, what is the probability that the cardinal

- a) is from the Americas?
 b) is over 70?
 c) is over 70 and from the Americas?
 d) is under 70 and not from Europe?

$$\begin{aligned}
 a) P(\text{is from Americas}) &= \frac{17+16}{115} = \frac{33}{115} & c) P(\text{is over 70 and from Americas}) &= \frac{16}{115} \\
 b) P(\text{is over 70}) &= \frac{34+16+11}{115} = \frac{61}{115} & d) P(\text{is under 70, not from Europe}) &= \frac{17+11}{115} = \frac{28}{115}
 \end{aligned}$$

2. (20pts) Write the probabilities and odds against and in favor of the following events (show any work needed below):

Event	probability	odds against	odds in favor
a) Rolling a 2 on a die	$\frac{1}{6}$	5:1	1:5
b) Drawing a 4 or 9 from a deck of cards	$\frac{8}{52} = \frac{2}{13}$	44:8 = 11:2	2:11
c) Drawing an ace or a red face card from a deck of cards	$\frac{10}{52} = \frac{5}{26}$	42:10 = 21:5	5:21
d) Getting numbers whose difference is 3 on a roll of two dice	$\frac{6}{36} = \frac{1}{6}$	30:6 = 5:1	1:5
e) Getting two heads in succession (anywhere) on three coin tosses	$\frac{3}{8}$	5:3	3:5

b) There are four 4's and four 9's in a deck

c) Four aces and six red face cards = 10

d) Possibilities
 $\left. \begin{array}{l} 1,4 \quad 4,1 \\ 2,5 \quad 5,2 \\ 3,6 \quad 6,3 \end{array} \right\} 6$

e) HHH •
 HHT •
 HTH
 HTT
 THH •
 THT
 TTH
 TTT

↑ ↓
 doesn't happen
 happens

3. (4pts) The odds in favor that a random college student will travel to a coastal location during spring break are 11-to-70.

$$11+70=81 \quad 11/81$$

a) What is the probability a student travels to a coastal location?

b) What is the probability a student doesn't travel to a coastal location?

$$70/81$$

4. (4pts) 74% of all gas station restrooms are reasonably clean. $100-74=26$

a) What are the odds in favor of choosing a gas station with a clean restroom?

$$74:26 = 37:13$$

b) What are the odds against choosing a gas station with a clean restroom?

$$26:74 = 13:37$$

5. (12pts) A coin is tossed five times in succession. Write the number of outcomes of this experiment and then compute the probability that you got

a) exactly one head?

b) a head on the second toss or a tail on the fourth toss?

$$\text{Total outcomes} = 2^5 = 32$$

c) at least one of the outcomes is a tail?

a) $P(\text{exactly one head}) = \frac{5}{32}$ (a single head can be on 1st, 2nd, 3rd, 4th or 5th toss - 5 ways to get only one head)

b) $P(\text{head on 2nd OR tail on 4th}) =$

$$= P(\text{head on 2nd}) + P(\text{tail on 4th}) - P(\text{head on 2nd AND tail on 4th})$$

choices: $\frac{1}{2} \frac{H}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2}$ $\frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{T}{2} \frac{1}{2}$ $\frac{1}{2} \frac{H}{2} \frac{1}{2} \frac{T}{2} \frac{1}{2}$

$$= \frac{16}{32} + \frac{16}{32} - \frac{16}{32} = \frac{16}{32} = \frac{1}{2} = 0.5$$

c) $P(\text{at least one is a tail}) = 1 - P(\text{all are heads}) = 1 - \frac{1}{32} = \frac{32}{32} - \frac{1}{32} = \frac{31}{32}$

$$= 0.96875$$

6. (10pts) Among 31 LEGO sets in a child's possession, 15 contain a green piece, 12 contain a wheel piece and 21 contain a green piece or a wheel. What is the probability that a randomly chosen set

a) contains both a green piece and a wheel?

b) neither contains a green piece nor a wheel?

c) $P(\text{green OR wheel}) = P(\text{green}) + P(\text{wheel}) - P(\text{green AND wheel})$

$$\frac{21}{31} = \frac{15}{31} + \frac{12}{31} - P(\text{green AND wheel})$$

$$\frac{21}{31} = \frac{27}{31} - P \quad \frac{21-27}{31} = -P \quad P(\text{green AND wheel}) = \frac{6}{31} = 0.193548$$

d) $P(\text{NOT (green OR wheel)}) = 1 - P(\text{green OR wheel}) = 1 - \frac{21}{31} = \frac{31}{31} - \frac{21}{31} = \frac{10}{31} = 0.322581$