$I = Prt$ $A = P(1 + rt)$ $A = P\left(1 + \frac{r}{n}\right)^{nt}$ $A = P\frac{\left(1+\frac{r}{n}\right)^{nt}-1}{\frac{r}{n}}$ $P = PMT\frac{1-\left(1+\frac{r}{n}\right)^{-nt}}{\frac{r}{n}}$ $Y = \left(1 + \frac{r}{n}\right)^{n} - 1$

1. (5pts) 13 is what percent of 45?

2. (6pts) You took out a loan with simple interest of 5% and repaid it with \$1253.18 after seven months. How much did you borrow?

3. (10pts) A jacket was initially sold for \$190, but it sold poorly, so the price was reduced 25%. After a while, sales picked up, so the store felt confident enough to raise the price 10%. What is the current price of the jacket?

4. (7pts) Cary deposited \$4500 in an account with 3.58% interest, compounded daily. How much is in the account in three years?

5. (14pts) In 2012, single mom Angela, who has one child, filed income taxes. Her total income was $73,000, she deposited $4,000 into a retirement account, paid $5,400 in mortgage interest, $1,200 in property taxes, $3,300 in state income taxes and donated $800 to charity. Use the table below to first determine Angela's taxable income (don't forget the exemptions) and then find the tax on this income.

Income bracket	Tax rate
up to $8,700	10%
$8,700–$35,350	15%
$35,350–$85,650	25%
$85,650–$178,650	28%
$178,650–$388,350	33%
more than $388,350	35%
exemption per person	$3,800
standard deduction	$5,950

6. (14pts) You would like to start a small lunch-serving business, so you save up to buy a food trailer, which costs $9,000.
a) How much should you deposit every month into an account with 3.75% interest, compounded monthly, in order to have $9,000 in three years?
b) How much of the final amount is from deposits and how much from interest?

7. (32pts) True story: the Duchess of Cambridge (aka Kate Middleton) is pregnant! Made-up part: since the family is expanding, the young royals will need a more family-oriented ride. They decide to purchase a luxury SUV costing $86,500 and finance the purchase with a 10-year loan at interest rate 4.35%, compounded monthly.

a) What is their montly payment on the loan?

b) What are their total payments over the course of the loan? How much of this amount is for interest?

c) How much of her first payment goes toward interest, and how much towards the principal?

d) How much do they owe after 6 years?

8. (12pts) An investment you are considering is expected to grow at rate 15% annually (compounded annually) for the foreseeable future. How long will it take until your investment quadruples?

Bonus. (10pts) To save for a car, Fulgencio initially deposited $3,000 into an account that gives 3.4% interest, compounded quarterly. Then, for the next three years, he deposited $1,000 every quarter into the same account.
a) How much is in the account after three years?
b) How much of the amount from a) is from deposits, and how much from interest?

$$\frac{a}{b} = \frac{P(E)}{1-P(E)} \qquad P(E) = \frac{a}{a+b} \text{ where odds in favor of } E \text{ are } a:b \qquad P(B\,|\,A) = \frac{n(A \text{ and } B)}{n(A)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \text{ or } B) = P(A) + P(B) \text{ (if } A \text{ and } B \text{ are mutually exclusive)}$$

$$P(A \text{ and } B) = P(A) \cdot P(B\,|\,A) \qquad P(A \text{ and } B) = P(A) \cdot P(B) \text{ if } A \text{ and } B \text{ are independent}$$

$$E = P_1 \cdot A_1 + P_2 \cdot A_2 + \cdots + P_n \cdot A_n$$

1. (6pts) A dating website classifies its members by race (four possibilities), eye color (five), hair color (seven), height (four brackets), and level of education (five). It claims that it has a member for evey possible variation of those features. If correct, how many members does the website have at least?

2. (6pts) Typical illnesses of children are: pink eye, ear infection, sore throat and cold. Suppose that over the course of four weeks, each of eight children was affected by exactly one disease, and a caretaker is assembling a report on who had what. How many different reports are possible?

3. (10pts) The table shows the number of employees at a small company with respect to gender and position. What is the probability, in fraction form, that a random employee:
a) is a manager?
b) is female?
c) is a male manager?
d) is an executive, given they are a man?
e) is a woman, given they are a worker?

Type	Man	Woman	Total
Executive	5	2	
Manager	10	6	
Worker	23	31	
Total			

4. (4pts) Suppose a new car has a 17% chance of needing repair in its first three years.
a) What are the odds in favor of the car needing repair?
b) What are the odds against the car needing repair?

5. (18pts) Write the probabilities and odds against and in favor of the following events (you can show any work needed below):

	Event	probability	odds against	odds in favor
a)	Getting an even number on a roll of a die			
b)	Drawing a black 2, 3, or 4 from a deck of cards			
c)	Getting exactly two heads on three coin tosses			
d)	Getting sum 4 or 6 on a roll of two dice			
e)	Getting exactly one tail on four coin tosses			

6. (10pts) Among a city's skyscrapers 23% have more than 70 floors, 42% are residential and 51% have more than 70 floors or are residential. If a skyscraper is randomly selected in this city, what is the probability it
a) has more than 70 floors and is residential?
b) lacks at least one of the features above?

7. (14pts) According to the MSU website, 60% of its students are female and 40% are male. An enterprising student has organized the following game outside of Winslow cafeteria: a player pays $2 and guesses the gender of the next person to come out. If they correctly guess it is a man, they win $4, if they correctly guess it is a woman, they win $3.
a) What is the expected value of the game if the player guesses a woman will come out next?
b) What is the expected value of the game if the player guesses a man will come out next?
c) If a player always guesses a woman comes out next, how much can they expect to win or lose after 20 plays?

8. (14pts) Suppose that 4% of expeditions to the peak of Mt. Everest result in a fatality. Assuming that having a fatality is independent among expeditions, what is the probability that
a) two expeditions both have a fatality?
b) four expeditions do not have a fatality?
c) there is a fatality on at least one of 10 expeditions?

9. (12pts) A spinner has 8 equal-size fields, one of which is labeled A, two are labeled R and five are labeled T. The letters resulting from three spins are used to spell a three-letter word. What is the probability that
a) the word is RAT?
b) the word ends with an R?

10. (6pts) If you draw two cards from a deck without replacement, what is the probability that are both face cards?

Bonus. (10pts) The game of chance described in problem 6 is played with the following twist: the player tosses a coin and lets that decide their guess, heads for woman, tails for man. With payouts as described in problem 6, what is the expected value of this game?

Final answers should have accuracy to 6 decimal places (or 4 decimal places for table-derived answers). Show some work how the mean and standard deviation are computed. *Giving only the answer will bring you few points.*

$$\text{midrange} = \frac{\text{lowest value} + \text{highest value}}{2} \qquad \text{range} = \text{highest value} - \text{lowest value}$$

$$\overline{x} = \frac{x_1 + x_2 + \cdots + x_n}{n} = \frac{\sum_i x_i}{n} = \frac{\sum_i x_i f_i}{n} \qquad Z = \frac{X - \overline{x}}{s}$$

$$s = \sqrt{\frac{(x_1 - \overline{x})^2 + (x_2 - \overline{x})^2 + \cdots + (x_n - \overline{x})^2}{n-1}} = \sqrt{\frac{\sum_i (x_i - \overline{x})^2}{n-1}} = \sqrt{\frac{\sum_i f_i (x_i - \overline{x})^2}{n-1}}$$

1. (8pts) A conservative candidate for state senator whose district mainly consists of one city wishes to gauge his overall popularity with voters. In order to find out, his team considers conducting a survey. Answer whether each of the following methods will produce a good, bad or questionable random sample of voters and comment why.

a □ good   Surveying students on a college campus.
   □ bad
   □ iffy

b □ good   Surveying random people from the city's voter lists.
   □ bad
   □ iffy

c □ good   Surveying Wal-Mart shoppers.
   □ bad
   □ iffy

d □ good   Surveying patrons at an upscale restaurant.
   □ bad
   □ iffy

2. (18pts) A dentist counts the number of cavities she fixes daily, and gets the numbers below.
a) Find the midrange.   0, 3, 7, 8, 4, 4, 5, 2, 3, 8, 0, 6, 4, 5
b) Find the median.
c) Find the mean.
d) Find the range.
e) Find the standard deviation.

3. (25pts) The owners of a mechanical bull are trying to determine whether their machine ejects the rider too soon. They measure the time that a rider was able to stay on the machine before getting ejected and come up with the data below (it shows that 6 riders stayed 3 seconds, 14 riders stayed 4 seconds, etc.)
a) Draw a histogram for the data.
b) Find the mode number of seconds on bull.
c) Find the median number of seconds on bull.
d) Find the mean number of seconds on bull.
e) Find the standard deviation.

time (seconds)	Frequency (riders)
3	6
4	14
5	13
6	11
7	8
8	3

4. (6pts) High school seniors Antonio and Berndt, from Italy and Germany, are both on track teams and compete in the 100-meter dash. The times for this dash for their age group are normally distributed with mean 12.3 seconds and standard deviation 0.7 for Italy, and mean 12.2 seconds and standard deviation 0.8 for Germany. At a track meet, Antonio ran the race in 11 seconds and Berndt ran the race in 10.8 seconds. Use z-scores to determine who did better relative to the runners in the same age group in their respective countries.

5. (14pts) The prices of new vehicles on a dealer's lot are given below in thousands.
a) Construct a grouped frequency distribution whose first class is 15-19.9.
b) Enter a representative value for each interval.
c) Use the representative values to estimate the mean of data. How does it compare to the actual mean of 25.9? (Do not compute the actual mean.)

21.6, 19.3, 15.7, 33.5, 37.8, 39.8, 25.6, 24.3, 31.4, 19.7, 17.1, 18.5, 27.6, 29.5, 30.4, 24.5, 24.6, 26.7, 28.8, 32.4, 16.9, 22.5, 20.4, 21.7, 35.6, 34.0, 17.5, 17.7, 19.2, 34.5, 25.5, 23.3, 29.5, 32.0

Class	Frequency	Representative Value

6. (12pts) Lengths of human pregnancies are normally distributed with a mean of 266 days and a standard deviation of 16 days. Use the 68-95-99.7 rule (draw a picture) to find the percentage of pregnancies that lasted

a) between 266 and 298 days

b) over 250 days

c) under 234 days

d) between 282 and 298 days

7. (17pts) The fuel consumption of midsize cards is normally distributed with mean 27.2 and standard deviation 4.7 (both in miles per gallon, MPG). Draw a picture showing which area you are computing as you answer:

a) What percentage of midsize cars have fuel consumption above 30 MPG?

b) What percentage of midsize cars have fuel consumption between 24 and 29 MPG?

Bonus. (10pts) A woman wishes to marry a man whose income is in at least the 75th percentile of earners. Assuming that incomes of eligible bachelors in her city are normally distributed with mean 47,000 and standard deviation 15,000, what is the least income her desired mate should possess? *(Hint: this problem is the inverse of what we usually do: an area is given and we have to find the z-score. Once you have the z-score, the income can be easily found)*

The rules: you may use your book and notes on this take-home exam. Your work is to be entirely your own: you may not talk to anybody else about the exam problems. If you are stuck with something, you may come to see me. Turn the exam in by Monday, May 6th, 3PM.

1. (30pts) A class of bored teenagers is trying to decide on what to use as a dare.[1] The choices are "jump from a tree", "drink cow urine", "sit in a restaurant refrigerator in summer clothes", "excavate a yellow jacket nest without a pesticide". Their preference rankings are shown below.

a) Which choice wins the vote in a plurality election?
b) Which choice wins the vote in a plurality election with elimination?
c) Which choice is the pairwise comparison winner?
d) Which choice is the winner using Borda's method? Perform the check on the sum of Borda points.

| Votes	6	7	4	2	8	9	3
1st	D	D	E	J	J	S	S
2nd	E	J	S	S	E	D	J
3rd	J	S	D	D	S	J	E
4th	S	E	J	E	D	E	D

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[1]Fictional morons. Do not attempt.

2. (17pts) Determine whether each of the following graphs has an Euler path or an Euler circuit. If it does, find it and state the order in which the vertices are visited, if not, explain why not.
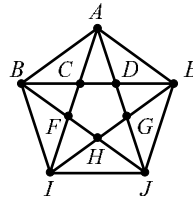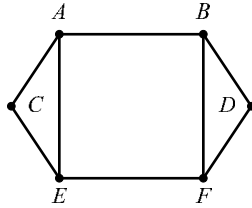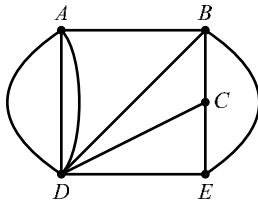
3. (13pts) Below is the floor plan of a house, with doors joining rooms indicated.
a) Represent the floor plan as a graph (rooms are vertices, don't forget an "outside").
b) Use the graph to determine if it is possible to walk around the house, passing through every door exactly once. If it is, draw the route.
c) Is it possible to do the same as in b), and start and finish in the same room?
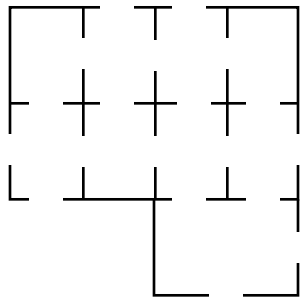
$I = Prt$ $A = P(1 + rt)$ $A = P\left(1 + \frac{r}{n}\right)^{nt}$ $A = P\frac{\left(1 + \frac{r}{n}\right)^{nt} - 1}{\frac{r}{n}}$

$P = PMT\frac{1 - \left(1 + \frac{r}{n}\right)^{-nt}}{\frac{r}{n}}$ $Y = \left(1 + \frac{r}{n}\right)^n - 1$

$\frac{a}{b} = \frac{P(E)}{1 - P(E)}$ $P(E) = \frac{a}{a+b}$ where odds in favor of E are $a : b$ $P(B \mid A) = \frac{n(A \text{ and } B)}{n(A)}$

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$P(A \text{ or } B) = P(A) + P(B)$ (if A and B are mutually exclusive)

$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$ $P(A \text{ and } B) = P(A) \cdot P(B)$ if A and B are independent

$\text{midrange} = \frac{\text{lowest value} + \text{highest value}}{2}$ $\text{range} = \text{highest value} - \text{lowest value}$

$\overline{x} = \frac{x_1 + x_2 + \cdots + x_n}{n} = \frac{\sum_i x_i}{n} = \frac{\sum_i x_i f_i}{n}$ $Z = \frac{X - \overline{x}}{s}$

$s = \sqrt{\frac{(x_1 - \overline{x})^2 + (x_2 - \overline{x})^2 + \cdots + (x_n - \overline{x})^2}{n - 1}} = \sqrt{\frac{\sum_i (x_i - \overline{x})^2}{n - 1}} = \sqrt{\frac{\sum_i f_i (x_i - \overline{x})^2}{n - 1}}$

$E = P_1 \cdot A_1 + P_2 \cdot A_2 + \cdots + P_n \cdot A_n$

1. (13pts) Assume the number of hours college students spend working per week is normally distributed with a mean of 15 hours and standard deviation 4 hours. Draw pictures showing which area you are computing as you answer:
a) What percentage of students work fewer than 8 hours per week?
b) What percentage of students work between 13 and 17 hours per week?

2. (12pts) Determine whether each of the following graphs has an Euler path or an Euler circuit. If it does, explain why, find it and state the order in which the vertices are visited, if not, explain why not.
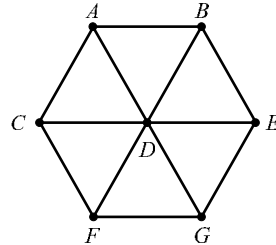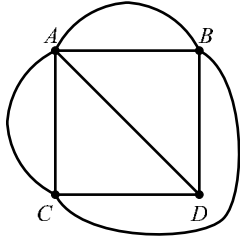
3. (23pts) Fans of the "Star Wars" saga were asked to elect their favorite episode of the original series (1977-1983). The rankings of the group are below.

Votes:	12	4	11	7	6	3
1st	IV	IV	V	V	VI	VI
2nd	V	VI	IV	VI	IV	V
3rd	VI	V	VI	IV	V	IV

a) Which choice wins the vote in a plurality election?
b) Which choice wins the vote in a plurality election with a runoff?
c) Which choice is the pairwise comparison winner?
d) Which choice is the winner using Borda's method? Perform the check on the sum of Borda points.

4. (25pts) The number of movie theaters in cities with populations greater than 10,000 across a US state is shown in the table below.
a) Draw a histogram for the data.
b) Find the mode number of movie theaters.
c) Find the median number of movie theaters.
d) Find the mean number of movie theaters.
e) Find the standard deviation.

Movie Theaters	Frequency (cities)
0	11
1	24
2	15
3	9
4	7
5	5

5. (13pts) Write the probabilities and odds against and in favor of the following events (you can show any work needed below):

Event	probability	odds against	odds in favor
a) Drawing a queen from a deck of cards			
b) Getting exactly one head on three coin tosses			
c) Getting sum more than 8 on a roll of two dice			

6. (12pts) A game of chance is set up like this: the player pays $10 and rolls a die. If the numbers 1 or 6 come up, the player wins $21, if 4 comes up, the player wins $15, otherwise the player wins nothing.
a) Find the expected value of this game.
b) What is the fair price of this game?
c) If a player played this game 100 times, how much would they expect to win or lose?

7. (5pts) In a subdivision of 43 houses, 13 have a pool, 21 have a three-car garage and 6 have both a pool and a three-car garage. If a home is randomly selected from the subdivision, what is the probability that it has a pool or a three-car garage?

8. (10pts) The probability that a student gets a job within a year after graduating is 75%. Assuming that different students getting jobs are independent events. What is the probability that:
a) Two students will get jobs after graduating?
b) At least one from a group of three will not get a job after graduating?

9. (7pts) If \$2,000 is deposited into an account bearing 2.55%, compounded daily, how much is in the account after two-and-a-half years?

10. (14pts) The Swokowskis would like to save up for a luxury car.
a) How much should they deposit every quarter into an account with 3.75% interest, compounded quarterly, in order to have \$35,000 in five years?
b) How much of the final amount is from deposits and how much from interest?

11. (16pts) The Middletons need to borrow $650,000 to help cover the cost of the wedding of their daughter. Suppose they can get a 10-year loan with interest rate 6%, compounded monthly.

a) What is their monthly payment?

b) What is the balance on the loan after 8 years?

Bonus. (15pts) Below is a floor plan of a building, with doors joining rooms indicated.

a) Represent the floor plan as a graph (don't forget to include an "outside").

b) Use the graph to determine if it is it possible to walk around the building, passing through every door exactly once. If it is, draw the route.

c) Is it possible to do the same as in b), and start and finish outside?