

Mathematical Concepts — Exam 3
MAT 117, Spring 2013 — D. Ivanšić

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 Show all your work!

Final answers should have accuracy to 6 decimal places (or 4 decimal places for table-derived answers). Show some work how the mean and standard deviation are computed. *Giving only the answer will bring you few points.*

$\text{midrange} = \frac{\text{lowest value} + \text{highest value}}{2}$	$\text{range} = \text{highest value} - \text{lowest value}$
$\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n} = \frac{\sum_i x_i}{n} = \frac{\sum_i x_i f_i}{n}$	$Z = \frac{X - \bar{x}}{s}$
$s = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \dots + (x_n - \bar{x})^2}{n - 1}} = \sqrt{\frac{\sum_i (x_i - \bar{x})^2}{n - 1}} = \sqrt{\frac{\sum_i f_i (x_i - \bar{x})^2}{n - 1}}$	

1. (8pts) A conservative candidate for state senator whose district mainly consists of one city wishes to gauge his overall popularity with voters. In order to find out, his team considers conducting a survey. Answer whether each of the following methods will produce a good, bad or questionable random sample of voters and comment why.

- a good Surveying students on a college campus.
 bad College students are generally anti-conservative, so would be biased against the congressman.
 iffy
- b good Surveying random people from the city's voter lists.
 bad It would ensure that everyone has an equal chance of being picked.
 iffy
- c good Surveying Wal-Mart shoppers.
 bad Almost everyone shops at WalMart, so it would not be a bad sample, but may skew toward the less-wealthy.
 iffy
- d good Surveying patrons at an upscale restaurant.
 bad The wealthier clientele of an upscale restaurant is usually more conservative, so may be biased in favor of the congressman.
 iffy

2. (18pts) A dentist counts the number of cavities she fixes daily, and gets the numbers below.

- a) Find the midrange.
- b) Find the median.
- c) Find the mean.
- d) Find the range.
- e) Find the standard deviation.

0, 3, 7, 8, 4, 4, 5, 2, 3, 8, 0, 6, 4, 5

1) 0, 0, 2, 3, 3, 4, 4, 4, 5, 5, 6, 7, 8, 8

14 items, need 7th & 8th, median = $\frac{4+4}{2} = \boxed{4}$

a) $\frac{0+8}{2} = 4$

c) $\bar{x} = \frac{0 \cdot 2 + 2 \cdot 1 + 3 \cdot 2 + 4 \cdot 3 + 5 \cdot 2 + 6 \cdot 1 + 7 \cdot 1 + 8 \cdot 2}{14}$

$= \frac{59}{14} = \boxed{4.214286}$

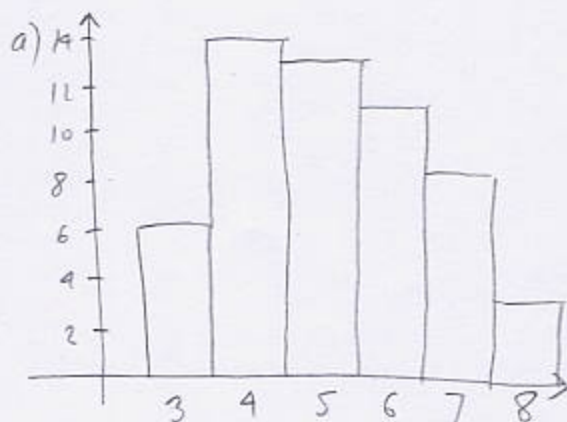
d) $2 \cdot (0 - 4)^2 + 1 \cdot (2 - 4)^2 + 2 \cdot (3 - 4)^2 + \dots = 84.35$

$s = \sqrt{\frac{84.35}{14-1}} = \sqrt{\frac{6.48}{11}} = \sqrt{0.589} = \boxed{2.547354}$

3. (25pts) The owners of a mechanical bull are trying to determine whether their machine ejects the rider too soon. They measure the time that a rider was able to stay on the machine before getting ejected and come up with the data below (it shows that 6 riders stayed 3 seconds, 14 riders stayed 4 seconds, etc.)

- Draw a histogram for the data.
- Find the mode number of seconds on bull.
- Find the median number of seconds on bull.
- Find the mean number of seconds on bull.
- Find the standard deviation.

time (seconds)	Frequency (riders)
3	6
4	14
5	13
6	11
7	8
8	3
	<u>55</u>



b.) mode = 4

c) 3, → 3, 4, → 4, 5, → 5, 6, → 6, 7, → 7, 8, → 8
6th 20th 33rd
 $55/2 = 27.5$, need 28th; 28th number is 5.

d)
$$\frac{3 \cdot 6 + 4 \cdot 14 + 5 \cdot 13 + 6 \cdot 11 + 7 \cdot 8 + 8 \cdot 3}{55} = \frac{285}{55} = 5.18181818$$

e)
$$6(3 - \quad)^2 + 14(4 - \quad)^2 + 13(5 - \quad)^2 + \dots + 3(8 - \quad)^2 = 106.1818$$

$$s = \sqrt{\frac{106.1818}{54}} = \sqrt{1.96\dots} = 1.402259$$

4. (6pts) High school seniors Antonio and Berndt, from Italy and Germany, are both on track teams and compete in the 100-meter dash. The times for this dash for their age group are normally distributed with mean 12.3 seconds and standard deviation 0.7 for Italy, and mean 12.2 seconds and standard deviation 0.8 for Germany. At a track meet, Antonio ran the race in 11 seconds and Berndt ran the race in 10.8 seconds. Use z-scores to determine who did better relative to the runners in the same age group in their respective countries.

$$z_1 = \frac{11 - 12.3}{0.7} \qquad z_2 = \frac{10.8 - 12.2}{0.8}$$

$$= -1.857143 \qquad = -1.75$$

Antonio is better with a lower z score (meaning faster!)

5. (14pts) The prices of new vehicles on a dealer's lot are given below in thousands.

a) Construct a grouped frequency distribution whose first class is 15-19.9.

b) Enter a representative value for each interval.

c) Use the representative values to estimate the mean of data. How does it compare to the actual mean of 25.9? (Do not compute the actual mean.)

21.6, 19.3, 15.7, 33.5, 37.8, 39.8, 25.6, 24.3, 31.4, 19.7, 17.1, 18.5, 27.6, 29.5, 30.4, 24.5, 24.6, 26.7, 28.8, 32.4, 16.9, 22.5, 20.4, 21.7, 35.6, 34.0, 17.5, 17.7, 19.2, 34.5, 25.5, 23.3, 29.5, 32.0

Class	Frequency	Representative Value
15-19.9	9	$17.45 = (15+19.9)/2$
20-24.9	8	22.45
25-29.9	7	27.45
30-34.9	7	32.45
35-39.9	3	37.45
	<u>34</u>	

$$\bar{x} \approx \frac{17.45 \cdot 9 + 22.45 \cdot 8 + \dots + 37.45 \cdot 3}{34}$$

$$= \frac{868.3}{34} = 25.538235$$

which is not too far from actual 25.9 (about 0.4 difference)

6. (12pts) Lengths of human pregnancies are normally distributed with a mean of 266 days and a standard deviation of 16 days. Use the 68-95-99.7 rule (draw a picture) to find the percentage of pregnancies that lasted

a) between 266 and 298 days

$$\frac{0.95}{2} = 0.475, \boxed{47.5\%}$$

b) over 250 days

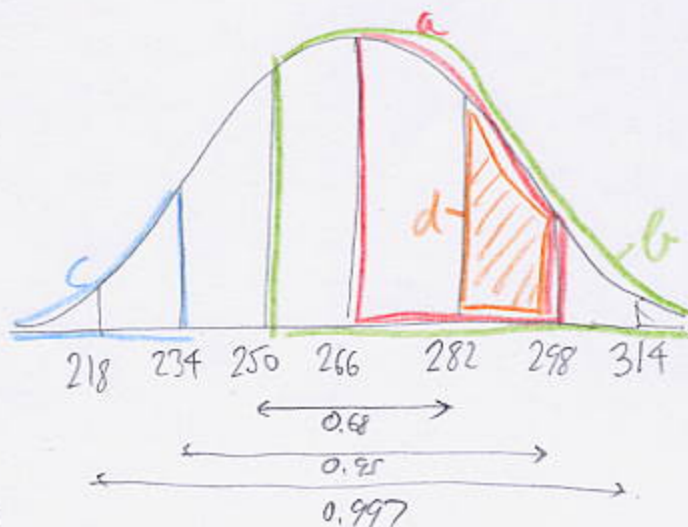
$$\frac{0.68}{2} + 0.5 = 0.84, \boxed{84\%}$$

c) under 234 days

$$0.5 - \frac{0.95}{2} = 0.5 - 0.475 = 0.025$$

d) between 282 and 298 days

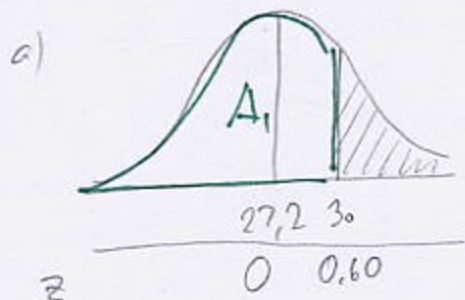
$$\frac{0.95}{2} - \frac{0.68}{2} = 0.475 - 0.34 = 0.135, \boxed{13.5\%}$$



7. (17pts) The fuel consumption of midsize cars is normally distributed with mean 27.2 and standard deviation 4.7 (both in miles per gallon, MPG). Draw a picture showing which area you are computing as you answer:

a) What percentage of midsize cars have fuel consumption above 30 MPG?

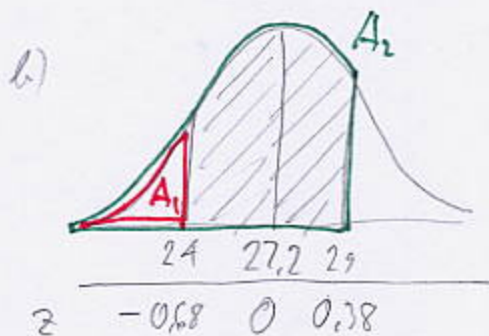
b) What percentage of midsize cars have fuel consumption between 24 and 29 MPG?



$$z = \frac{30 - 27.2}{4.7} = 0.5957\dots$$

$$\text{shaded area} = 1 - \overbrace{0.7257}^{A_1} = 0.2743$$

$$\boxed{27.43\%}$$



$$z_1 = \frac{24 - 27.2}{4.7} = -0.6808\dots$$

$$z_2 = \frac{29 - 27.2}{4.7} = 0.3829\dots$$

$$\text{shaded} = A_2 - A_1 = 0.6480 - 0.2483 = 0.3997$$

$$\boxed{39.97\%}$$

Bonus. (10pts) A woman wishes to marry a man whose income is in at least the 75th percentile of earners. Assuming that incomes of eligible bachelors in her city are normally distributed with mean 47,000 and standard deviation 15,000, what is the least income her desired mate should possess? (Hint: this problem is the inverse of what we usually do: an area is given and we have to find the z-score. Once you have the z-score, the income can be easily found)



Need z so that area is 0.75.

In the table, the closest area to 0.75

is 0.7486, which occurs for $z = 0.67$

Mate's income needs to be at least 0.67 standard deviations above mean, so $47 + 0.67 \cdot 15 = 57.05$, so \$57,050.