Mathematical Concepts — Exam 2
MAT 117, Spring 2013 — D. Ivanšić

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Show all your work!

$\frac{a}{b} = \frac{P(E)}{1-P(E)}$ $\quad P(E) = \frac{a}{a+b}$ where odds in favor of E are $a : b$ $\quad P(B \mid A) = \frac{n(A \text{ and } B)}{n(A)}$

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$P(A \text{ or } B) = P(A) + P(B)$ (if A and B are mutually exclusive)

$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$ $\quad P(A \text{ and } B) = P(A) \cdot P(B)$ if A and B are independent

$E = P_1 \cdot A_1 + P_2 \cdot A_2 + \cdots + P_n \cdot A_n$

1. (6pts) A dating website classifies its members by race (four possibilities), eye color (five), hair color (seven), height (four brackets), and level of education (five). It claims that it has a member for evey possible variation of those features. If correct, how many members does the website have at least?

race eye hair height edu

choices 4 5 7 4 5

$4 \cdot 5 \cdot 7 \cdot 4 \cdot 5 = 2800$

At least 2800 members

2. (6pts) Typical illnesses of children are: pink eye, ear infection, sore throat and cold. Suppose that over the course of four weeks, each of eight children was affected by exactly one disease, and a caretaker is assembling a report on who had what. How many different reports are possible?

4 4 4 4 4 4 4 4 choices

$4 \cdot 4 \cdots 4 = 4^8 = 65,536$ reports

3. (10pts) The table shows the number of employees at a small company with respect to gender and position. What is the probability, in fraction form, that a random employee:

a) is a manager?
b) is female?
c) is a male manager?
d) is an executive, given they are a man?
e) is a woman, given they are a worker?

Type	Man	Woman	Total
Executive	5	2	7
Manager	10	6	16
Worker	23	31	54
Total	38	39	77

a) $\frac{16}{77}$ b) $\frac{39}{77}$ c) $\frac{10}{77}$ d) $\frac{5}{38}$ e) $\frac{31}{54}$

100 - 17 = 83

4. (4pts) Suppose a new car has a 17% chance of needing repair in its first three years.
a) What are the odds in favor of the car needing repair? 17 to 83
b) What are the odds against the car needing repair? 83 to 17

5. (18pts) Write the probabilities and odds against and in favor of the following events (you can show any work needed below):

	Event	probability	odds against	odds in favor
a)	Getting an even number on a roll of a die	$\frac{3}{6} = \frac{1}{2}$	3:3 = 1:1	1:1
b)	Drawing a black 2, 3, or 4 from a deck of cards	$\frac{6}{52} = \frac{3}{26}$	23:3	3:23
c)	Getting exactly two heads on three coin tosses	$\frac{3}{8}$	5:3	3:5
d)	Getting sum 4 or 6 on a roll of two dice	$\frac{8}{36} = \frac{2}{9}$	28:8 = 7:2	2:7
e)	Getting exactly one tail on four coin tosses	$\frac{4}{16} = \frac{1}{4}$	12:4 = 3:1	1:3

a) 2, 4, 6 - 3 numbers out of 6

b) (2, 3, 4) × 2 (black) 16 cards out of 52

c) HHH
HHT
HTH
HTT
THH
THT
TTH
TTT

d) 4 = 1+3, 2+2, 3+1
6 = 1+5, 2+4, 3+3, 4+2, 5+1
8 possibilities
36 outcomes

e) _ _ _ _ 16 outcomes
Exactly one tail occurs on 1st, 2nd, 3rd or 4th toss (all others heads)
— 4 possibilities

6. (10pts) Among a city's skyscrapers 23% have more than 70 floors, 42% are residential and 51% have more than 70 floors or are residential. If a skyscraper is randomly selected in this city, what is the probability it
a) has more than 70 floors and is residential?
b) lacks at least one of the features above?

a) P(>70 floors OR residential) = P(>70 floors) + P(residential) − P(>70 floors AND resident

$0.51 = 0.23 + 0.42 - P \quad | +P - 0.51$

$P = 0.23 + 0.42 - 0.51 = 0.14 = P(>70 \text{ floors AND residential})$

b) lacks at least one = doesn't have both

P(doesn't have(>70 floors AND residential)) = 1 − P(>70 floors AND residential)

$= 1 - 0.14 = 0.86$

7. (14pts) According to the MSU website, 60% of its students are female and 40% are male. An enterprising student has organized the following game outside of Winslow cafeteria: a player pays \$2 and guesses the gender of the next person to come out. If they correctly guess it is a man, they win \$4, if they correctly guess it is a woman, they win \$3.

a) What is the expected value of the game if the player guesses a woman will come out next?

b) What is the expected value of the game if the player guesses a man will come out next?

c) If a player always guesses a woman comes out next, how much can they expect to win or lose after 20 plays?

a) outcomes:

	net win	prob.
woman	3-2=1	0.6
man	-2	0.4

$E = 0.6 \cdot 1 + 0.4(-2)$

$= 0.6 - 0.8 = -0.20$

expect to lose 20c per game

b) outcomes:

	net win	prob.
woman	-2	0.6
man	4-2=2	0.4

$E = 0.6 \cdot (-2) + 0.4 \cdot 2 =$

$= -1.20 + 0.8 = -0.40$

expect to lose 40c per game

c) $20 \cdot (-0.20) = -4$ Player may expect to lose \$4,

8. (14pts) Suppose that 4% of expeditions to the peak of Mt. Everest result in a fatality. Assuming that having a fatality is independent among expeditions, what is the probability that

a) two expeditions both have a fatality?

b) four expeditions do not have a fatality?

c) there is a fatality on at least one of 10 expeditions?

a) $P((\text{Fatality on 1st}) \text{ AND } (\text{Fatality on 2nd})) = P(\text{Fatality on 1st}) \cdot P(\text{Fatality on 2nd})$

$= 0.04 \cdot 0.04 = 0.0016$

b) $P((\text{no fat. on 1st}) \text{ AND } (\text{no fat. on 2nd}) \text{ AND } (\text{no fat. on 3rd}) \text{ AND } (\text{no fat. on 4th}))$

$= P(\text{no fat. on 1st}) \cdot P(\text{no fat. on 2nd}) \cdots = 0.96 \cdot 0.96 \cdot 0.96 \cdot 0.96$

$= 0.96^4 = 0.849347$

c) $P(\text{fat. on at least one}) = 1 - P(\text{no fatality on any})$

$= 1 - P((\text{no fat on 1st}) \text{ AND } \_ \text{ AND } (\text{no fat. on 10th}))$

$= 1 - 0.96 \cdot 0.96 \cdots 0.96 = 1 - 0.96^{10} = 1 - 0.66\_ = 0.335167$

9. (12pts) A spinner has 8 equal-size fields, one of which is labeled A, two are labeled R and five are labeled T. The letters resulting from three spins are used to spell a three-letter word. What is the probability that

a) the word is RAT?

b) the word ends with an R?

A -- 1
R -- 2
T -- 5

a) $P((R \text{ on 1st}) \text{ AND } (A \text{ on 2nd}) \text{ AND } (T \text{ on 3rd}))$
$= P(R \text{ on 1st}) \cdot P(A \text{ on 2nd}) \cdot P(T \text{ on 3rd}) = \frac{2}{8} \cdot \frac{1}{8} \cdot \frac{5}{8} = \frac{5}{256} = 0.0195313$

b) $P((\text{any on 1st}) \text{ AND } (\text{any on 2nd}) \text{ AND } (R \text{ on 3rd}))$
$= P(\text{any}) \cdot P(\text{any}) \cdot P(R \text{ on 3rd}) = 1 \cdot 1 \cdot \frac{2}{8} = \frac{1}{4} = 0.25$

10. (6pts) If you draw two cards from a deck without replacement, what is the probability that are both face cards?

$P(\text{1st face AND 2nd face}) = P(\text{1st face}) \cdot P(\text{2nd face} \mid \text{1st face})$
$= \frac{12}{52} \cdot \frac{11}{51} = \frac{11}{221} = 0.0497738$

face cards: 12

Bonus. (10pts) The game of chance described in problem 6 is played with the following twist: the player tosses a coin and lets that decide their guess, heads for woman, tails for man. With payouts as described in problem 6, what is the expected value of this game?

Outcomes	toss	gender exits	net win	prob.
	H AND W		3-2=1	$0.5 \cdot 0.6 = 0.3$
	H AND M		-2	$0.5 \cdot 0.4 = 0.2$
	T AND W		-2	$0.5 \cdot 0.6 = 0.3$
	T AND M		4-2=2	$0.5 \cdot 0.4 = 0.2$

$E = 0.3 \cdot 1 + 0.2(-2) + 0.3(-2) + 0.2 \cdot 2 = 0.3 - 0.4 - 0.6 + 0.4 = -0.30$

expect to lose 30¢ per game.