

$$I = Prt \quad A = P(1 + rt) \quad A = P \left(1 + \frac{r}{n}\right)^{nt} \quad A = P \frac{\left(1 + \frac{r}{n}\right)^{nt} - 1}{\frac{r}{n}} \quad P = PMT \frac{1 - \left(1 + \frac{r}{n}\right)^{-nt}}{\frac{r}{n}} \quad Y = \left(1 + \frac{r}{n}\right)^n - 1$$

1. (5pts) 13 is what percent of 45?

$$A = P \cdot B \quad P = \frac{13}{45} = 0.288889$$
$$13 = P \cdot 45 \quad 28.8889\%$$

2. (6pts) You took out a loan with simple interest of 5% and repaid it with \$1253.18 after seven months. How much did you borrow?

$$A = P(1 + rt) \quad 1253.18 = P \cdot 1.0291\bar{6}$$
$$1253.18 = P \left(1 + 0.05 \cdot \frac{7}{12}\right) \quad P = \frac{1253.18}{1.0291\bar{6}} = 1217.66$$

3. (10pts) A jacket was initially sold for \$190, but it sold poorly, so the price was reduced 25%. After a while, sales picked up, so the store felt confident enough to raise the price 10%. What is the current price of the jacket?

OK,

$$0.25 \cdot 190 = 47.50$$
$$190 - 47.50 = 142.50$$
$$142.50 \cdot 0.1 = 14.25$$
$$142.50 + 14.25 = 156.75$$
$$\text{last price} = 1.1 \cdot 0.75 \text{ old price} = 156.75$$

4. (7pts) Cary deposited \$4500 in an account with 3.58% interest, compounded daily. How much is in the account in three years?

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$
$$A = 4500 \left(1 + \frac{0.0358}{365}\right)^{365 \cdot 3} = 4500 \cdot 1.11\bar{6} = 5010.18$$

5. (14pts) In 2012, single mom Angela, who has one child, filed income taxes. Her total income was \$73,000, she deposited \$4,000 into a retirement account, paid \$5,400 in mortgage interest, \$1,200 in property taxes, \$3,300 in state income taxes and donated \$800 to charity. Use the table below to first determine Angela's taxable income (don't forget the exemptions) and then find the tax on this income.

Income bracket	Tax rate
up to \$8,700	10%
\$8,700-\$35,350	15%
\$35,350-\$85,650	25%
\$85,650-\$178,650	28%
\$178,650-\$388,350	33%
more than \$388,350	35%
exemption per person	\$3,800
standard deduction	\$5,950

$$\begin{aligned} \text{deductions} &= 5400 + 1200 + 3300 + 800 \\ &= 10,700, \text{ greater than standard deduction} \\ &\quad (\text{so use this number}) \end{aligned}$$

$$\begin{aligned} \text{Taxable income} &= 73,000 - (4,000 + 10,700 + 3,800 \cdot 2) \\ &= 73,000 - 22,300 = 50,700 \end{aligned}$$

$$\begin{aligned} \text{Tax} &= 0.10 \cdot 8700 + 0.15 \cdot (35,350 - 8700) + 0.25 \cdot (50,700 - 35,350) \\ &= 870 + 3997.50 + 3837.50 \\ &= 8705 \end{aligned}$$

6. (14pts) You would like to start a small lunch-serving business, so you save up to buy a food trailer, which costs \$9,000.

- a) How much should you deposit every month into an account with 3.75% interest, compounded monthly, in order to have \$9,000 in three years?  
 b) How much of the final amount is from deposits and how much from interest?

$$\begin{aligned} a) \quad A &= P \frac{(1 + \frac{r}{n})^{nt} - 1}{\frac{r}{n}} \\ 9000 &= P \frac{(1 + \frac{0.0375}{12})^{12 \cdot 3} - 1}{\frac{0.0375}{12}} \end{aligned}$$

$$9000 = P \cdot 38.040 \dots$$

$$P = \frac{9000}{38.040} = 236.59$$

$$\begin{aligned} b) \quad 236.59 \cdot 12 \cdot 3 &= 8517.24 \text{ From deposits} \\ 9000 - 8517.24 &= 482.76 \end{aligned}$$

7. (32pts) True story: the Duchess of Cambridge (aka Kate Middleton) is pregnant! Made-up part: since the family is expanding, the young royals will need a more family-oriented ride. They decide to purchase a luxury SUV costing \$86,500 and finance the purchase with a 10-year loan at interest rate 4.35%, compounded monthly.

- What is their monthly payment on the loan?
- What are their total payments over the course of the loan? How much of this amount is for interest?
- How much of her first payment goes toward interest, and how much towards the principal?
- How much do they owe after 6 years?

$$a) P = PMT \frac{1 - (1 + \frac{r}{n})^{-nt}}{\frac{r}{n}}$$

$$86,500 = PMT \cdot \frac{1 - (1 + \frac{0.0435}{12})^{-12 \cdot 10}}{\frac{0.0435}{12}}$$

$$86,500 = PMT \cdot 97.16...$$

$$PMT = \frac{86,500}{97.16} = 890.23$$

$$1) \text{ Total payments} = 890.23 \cdot 12 \cdot 10 =$$

$$= 106,827.60$$

$$\text{Total interest paid} = 106,827.60 - 86,500$$

$$= 20,327.60$$

$$c) 86,500 \cdot 0.0435 \cdot \frac{1}{12} = 313.56$$

toward interest

$$890.23 - 313.56 = 576.67$$

$$d) P = PMT \frac{1 - (1 + \frac{r}{n})^{-nt}}{\frac{r}{n}}$$

$$P = 890.23 \frac{1 - (1 + \frac{0.0435}{12})^{-12 \cdot 4}}{\frac{0.0435}{12}}$$

$$= 890.23 \cdot 43.98...$$

$$= 39,155.06$$

8. (12pts) An investment you are considering is expected to grow at rate 15% annually (compounded annually) for the foreseeable future. How long will it take until your investment quadruples?

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

$$4P = P \left(1 + \frac{0.15}{1}\right)^{1 \cdot t} \quad | \div P$$

$$4 = (1.15)^t \quad | \log$$

$$\log 4 = \log (1.15)^t$$

$$\log 4 = t \log (1.15)$$

$$t = \frac{\log 4}{\log 1.15} = 9.918969$$

About 9.92 years

**Bonus.** (10pts) To save for a car, Fulgencio initially deposited \$3,000 into an account that gives 3.4% interest, compounded quarterly. Then, for the next three years, he deposited \$1,000 every quarter into the same account.

a) How much is in the account after three years?

b) How much of the amount from a) is from deposits, and how much from interest?

Treat as two accounts

From 3000:

$$A = 3000 \left(1 + \frac{0.034}{4}\right)^{4 \cdot 3}$$

$$= 3000 \cdot 1.1069 \dots$$

$$= 3320.72$$

From quarterly deposits:

$$A = 1000 \frac{\left(1 + \frac{0.034}{4}\right)^{4 \cdot 3} - 1}{\frac{0.034}{4}}$$

$$= 1000 \cdot 12.57 \dots$$

$$= 12,577.20$$

Combined amount: 15,897.92

b) Total deposits:

$$3000 + 1000 \cdot 4 \cdot 3 = 15000$$

From interest:

$$15,897.92 - 15,000 = 897.92$$