Calculus 3 — Final Exam **MAT 309, Fall 2012 — D. Ivanšić**

Name: ______________________________

Show all your work!

1. (6pts) What is the angle between vectors $\mathbf{a}$ and $\mathbf{b}$ if we know that $||\mathbf{a}|| = 3$, $||\mathbf{b}|| = 4$ and $\mathbf{a} \cdot \mathbf{b} = 6$?

2. (10pts) Find the equation of the plane that contains the points $A = (1, 3, 1)$, $B = (4, -7, 2)$ and $C = (-1, -1, -1)$.

3. (16pts) Consider the function $f(x, y) = \dfrac{y}{x}$ for $x, y > 0$.
c) Draw a rough contour map for the function, with contour interval $\frac{1}{4}$, going from $c = \frac{1}{4}$ to $c = \frac{3}{2}$.
b) Find ∇f and roughly draw this vector field (scale the vectors for a better picture) Note that no computation is needed to draw the vector field.
c) What is $\int_C \nabla f \cdot d\mathbf{s}$ if C is the arc of the parabola $y = x^2$ from $(1, 1)$ to $(3, 9)$?

4. (14pts) A curve is given by $\mathbf{r}(t) = \langle \cos t, \sin t, 5t \rangle$, $t \in [0, 4\pi]$.
a) Sketch this curve.
b) Find the parametric equation of the tangent line to the curve at time $t = \pi$ and draw this tangent line on your sketch.

5. (10pts) Find the equation of the tangent plane to the surface $\dfrac{x^2}{12} - \dfrac{y^2}{4} + \dfrac{z^2}{2} = 1$ at the point $(3, 1, -1)$. Simplify the equation to standard form.

6. (14pts) Let $f(x,y) = \dfrac{e^{xy}}{x^2}$, $x = -2u + 5v$, $y = u - 3v$. Use the chain rule to find $\dfrac{\partial f}{\partial u}$ when $u = 6$, $v = 2$.

7. (16pts) Find and classify the local extremes for $f(x,y) = 2x^2y^2 + 3x^3 - 4x$.

8. (18pts) Find $\iint_D x^2 + y^2 \, dA$ if D is the region above $y = |x|$ and between lines $y = 3$ and $y = 5$. Sketch the region of integration.

9. (12pts) Sketch the region W that is the part of the ball $x^2 + y^2 + z^2 \leq 16$, above the plane $z = 3$, and to the right of the plane $y = 2$. Then write the iterated triple integral that stands for $\iiint_W f \, dV$ which ends in $dy \, dz \, dx$.

10. (22pts) Compute the integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$, if S is the part of the paraboloid $z = 10 - x^2 - y^2$ that is above the xy-plane and $\mathbf{F}(x, y, z) = \langle x, y, 1 + z \rangle$. (The surface does not include any part of the xy-plane, just part of the paraboloid.) Use the normal vectors to the paraboloid that point upwards. Draw the surface and some normal vectors, parametrize the surface and specify the planar region D where your parameters come from.

11. (12pts) Is the vector field below a gradient field? If yes, find its potential function.

$\mathbf{F}(x, y, z) = \langle -z^3, 3z, 2z + 3y - 3xz^2 \rangle$

Bonus. (15pts) The intersection of balls $x^2 + y^2 + z^2 \leq 1$ and $x^2 + y^2 + (z-1)^2 \leq 1$ is a lens-shaped region. Find its volume by doing the following:
a) Use spherical coordinates to find the volume of the region inside $x^2 + y^2 + (z-1)^2 \leq 1$ that is outside of $x^2 + y^2 + z^2 \leq 1$.
b) Find the volume of the lens using your result from a). Recall that the volume of a ball of radius R is $\frac{4}{3}\pi R^3$.