Calculus 3 — Exam 5
MAT 309, Fall 2012 — D. Ivanšić

Name: ______________________________

Show all your work!

1. (15pts) Let $\mathbf{F}(x, y) = \langle y, x \rangle$.
a) Roughly draw the vector field $\mathbf{F}(x, y)$, scaling the vectors for a better picture.
b) Guess a function $\phi(x, y)$ so that $\mathbf{F} = \nabla\phi$.
c) How could you have roughly done a) without evaluating the vector field at various points?
d) What is $\int_C \mathbf{F} \cdot d\mathbf{s}$ if C is part of the curve $y = \sin x$ from $(0, 0)$ to $(\frac{\pi}{2}, 1)$? How about if C is a straight line segment from $(0, 0)$ to $(\frac{\pi}{2}, 1)$?
e) What is $\int_C \mathbf{F} \cdot d\mathbf{s}$ if C is the unit circle?

2. (15pts) Let C be the curve $x = 1 + t$, $y = 4 \sin t$, $z = t^2$, for $t \in [0, \pi]$.
a) Set up $\int_C z(e^x + e^y)\, ds$.
b) Set up $\int_C \mathbf{F} \cdot d\mathbf{s}$, if $\mathbf{F}(x, y, z) = \langle x^2, z, y^2 \rangle$.
In both cases simplify the set-up, but do not evaluate the integral.

3. (16pts) One of the two vectors fields below is not a gradient field, and the other one is (cross partials, remember?). Identify which is which, and find the potential function for the one that is.

$\mathbf{F}(x, y, z) = \langle \cos(xz), \sin(yz), xy \sin z \rangle$ $\qquad$ $\mathbf{G}(x, y, z) = \langle 2xy + z^2, x^2 + 2yz, y^2 + 2xz \rangle$

4. (12pts) A surface is parametrized by $\Phi(u, v) = (u^2 + v^2, uv, u^2 - v^2)$. Find the equation of the tangent plane to this surface at the point where $(u, v) = (1, 1)$.

5. (24pts) Find the surface integral $\iint_S x\, dS$, if S is part of the sphere $x^2 + y^2 + z^2 = 4$ that is in the octant $x, y, z \geq 0$. Draw the surface, parametrize it and specify the planar region D where your parameters come from.

6. (18pts) Set up the integral for $\iint_S \mathbf{F} \cdot d\mathbf{S}$, if S is the part of the paraboloid $z = 10 - x^2 - y^2$ that is above the xy-plane and $\mathbf{F}(x, y, z) = \langle x, y, 1 + z \rangle$. (The surface does not include any part of the xy-plane, just part of the paraboloid.) Use the normal vectors to the paraboloid that point upwards. Draw the surface and some normal vectors, parametrize the surface and specify the planar region D where your parameters come from. Simplify the set-up, but do not evaluate the integral.

Bonus. (10pts) Find the surface area of the part of the sphere $x^2 + y^2 + z^2 = 9$ that is between the planes $z = 1$ and $z = 2$.