Calculus 3 — Exam 4
MAT 309, Fall 2012 — D. Ivanšić

Name: _____

Show all your work!

1. (18pts) Find $\iint_D x + y \, dA$ if D is the triangle with vertices $(0,0)$, $(1,1)$, $(2,1)$. Sketch the region of integration.

2. (18pts) Evaluate $\int_0^2 \int_0^{\sqrt{4-x^2}} (4-y^2)^{\frac{3}{2}} \, dy \, dx$ by changing the order of integration. Sketch the region of integration.

3. (16pts) Use polar coordinates to find the area of the region that is inside the unit circle and to the right of the line $x = \frac{1}{2}$. Sketch the region of integration first.

4. (16pts) Sketch the region W that is in the first octant $(x, y, z \geq 0)$, above the plane $y = z$ and below the plane $x + y + z = 1$. Then write the two iterated triple integrals that stand for $\iiint_W f \, dV$ which end in $dx\, dz\, dy$, and $dz\, dy\, dx$.

5. (16pts) Use cylindrical coordinates to set up $\iiint_W \frac{x+y+z}{x^2+y^2}\,dV$ where W is the part of the region above the paraboloid $z = x^2 + y^2$ and below the sphere $x^2 + y^2 + z^2 = 12$ where $x \geq 0$. Sketch the region of integration. Do not evaluate the integral.

6. (16pts) Use change of variables to find the integral $\iint_D (2x+y)^2\,dA$ if D is the rhombus bounded by $y = 2x + 2$, $y = 2x - 2$, $y = -2x + 2$, $y = -2x - 2$. Sketch the region D.

Bonus. (10pts) The intersection of balls $x^2 + y^2 + z^2 \leq 1$ and $x^2 + y^2 + (z-1)^2 \leq 1$ is a lens-shaped region. Find its volume by doing the following:
a) Use spherical coordinates to find the volume of the region inside $x^2 + y^2 + (z-1)^2 \leq 1$ that is outside of $x^2 + y^2 + z^2 \leq 1$.
b) Find the volume of the lens using your result from a). Recall that the volume of a ball of radius R is $\frac{4}{3}\pi R^3$.