Calculus 3 — Exam 3 **MAT 309, Fall 2012 — D. Ivanšić**

Name:______________________________

Show all your work!

1. (12pts) Find the equation of the tangent plane to the surface $x^2 - \frac{y^2}{4} - \frac{z^2}{9} = 1$ at the point $\left(2, \sqrt{2}, 3\sqrt{\frac{5}{2}}\right)$. Simplify the equation to standard form.

2. (18pts) Let $f(x, y) = x^2 - y^2$.
a) Find the directional derivative of f at the point $(3, 2)$ in the direction of $\mathbf{v} = \langle 1, -5 \rangle$.
b) At the point $(3, 2)$, in which direction does the function increase the fastest? Decrease the fastest?
c) Find $\frac{d}{dt} f(\mathbf{r}(t))$ for the path $\mathbf{r}(t) = \langle 4 - 3t, t^2 - 2t \rangle$ when $t = 1$.

3. (20pts) Let $f(x,y) = \dfrac{\ln x}{\ln x + \ln y}$, $x = e^u \cos v$, $y = e^u \sin v$. Use the chain rule to find $\dfrac{\partial f}{\partial v}$ when $u = 3$, $v = \dfrac{\pi}{6}$.

4. (14pts) In an improbable scenario, you find yourself on a desert island with no technology and have to estimate $\dfrac{\sqrt{20.2}}{\sqrt{4.99}}$. (Or, in a more likely one, you find yourself in test-taking environment with no calculators allowed.) Use linearization to estimate this number, and compare it to the calculator result of 2.011988.

5. (14pts) Using implicit differentiation, find $\dfrac{\partial z}{\partial y}$ at the point $(-1, 2, 1)$, if $x^2y+y^2z+z^2x = 5$.

6. (22pts) Find and classify the local extremes for $f(x, y) = \frac{2}{3}x^3 + 2xy - 2y^2 - 10x$.

Bonus (10pts) Find the absolute maximum and minimum of $f(x, y) = x^2 + y^2 - 6x - 4y$ on the domain $x \geq 0$, $y \geq 0$, $y \leq -2x + 3$.