Calculus 3 — Exam 2
MAT 309, Fall 2012 — D. Ivanšić

Name: ________________________

Show all your work!

1. (14pts) A curve is given by $\mathbf{r}(t) = \left\langle \cos t, \sin t, \sin \frac{t}{2} \right\rangle$, $t \in [0, 2\pi]$.
a) Sketch this curve.
b) Find the parametric equation of the tangent line to the curve at time $t = \frac{\pi}{2}$ and draw this tangent line on your sketch.

2. (16pts) Consider the curve C that is the intersection of the cylinder $x^2 + z^2 = 4$ with the parabolic cylinder $z = y^2$.
a) Sketch a picture.
b) Parametrize each of the two parts of the curve corresponding to $x \geq 0$ and $x \leq 0$, taking y as the parameter.
c) What interval does the parameter run through to get each of the two parts?

3. (22pts) Consider the function $f(x, y) = \dfrac{y}{x^2}$ for $x > 0$, y any.

a) Identify and draw vertical traces for this function.

b) Using a), draw the graph of the function.

c) Draw a rough contour map for the function, with contour interval $\frac{1}{2}$, going from $c = -\frac{3}{2}$ to $c = \frac{3}{2}$.

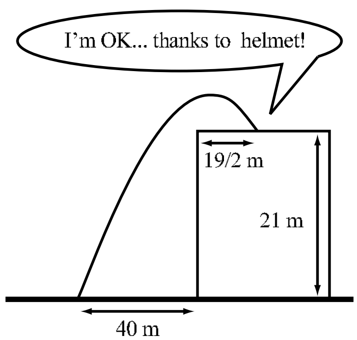
d) By looking at the contour map, indicate a path on which we could move from $(\sqrt{2}, 1)$ in order to increase the value of the function to 1.

4. (16pts) Find the length of the curve with the parametrization $\mathbf{r}(t) = \left\langle t^{\frac{3}{2}}, 5\sin t, 5\cos t \right\rangle$, $t \in [0, 4\pi]$.

5. (20pts) Acting on a dare, your favorite physics professor Dr. __________ (insert name here) launches himself from 40 meters away from base of Faculty Hall (height 21 meters) and lands on its roof $\frac{19}{2}$ meters away from the edge. (See picture.) The angle α of launch was such that $\cos\alpha = \frac{3}{5}$. Assume $g = 10$ for simplicity.

a) Find his position at time t. The expression will have an unknown initial speed v_0 in it.

b) Now find v_0.



6. (12pts) Determine and sketch the domain of the function $f(x, y) = \dfrac{\sqrt{x - y - 5}}{x + y}$.

Bonus (10pts) Use coordinates to prove the formula $(\mathbf{u}(t) \times \mathbf{v}(t))' = \mathbf{u}'(t) \times \mathbf{v}(t) + \mathbf{u}(t) \times \mathbf{v}'(t)$ for any two vector functions $\mathbf{u}(t)$ and $\mathbf{v}(t)$,