Calculus 3 — Exam 1
MAT 309, Fall 2012 — D. Ivanšić

Name: _____

Show all your work!

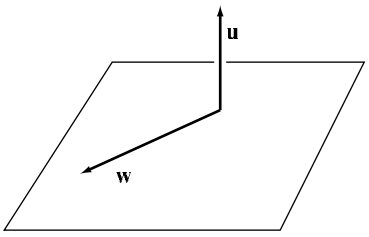
1. (18pts) Let $\mathbf{u} = \langle 3, 1, -3 \rangle$ and $\mathbf{v} = \langle 0, 2, -1 \rangle$.
a) Calculate $2\mathbf{u}$, $4\mathbf{u} - 3\mathbf{v}$, and $||\mathbf{u}||$.
b) Find the unit vector in direction of $\mathbf{v}$.
c) Find the projection of $\mathbf{u}$ onto $\mathbf{v}$.

2. (4pts) Do the coordinate systems given by the sets of vectors below (in order listed) satisfy the right hand rule?

$\{\mathbf{j}, \mathbf{k}, \mathbf{i}\}$

$\{\mathbf{i}, \mathbf{k}, \mathbf{j}\}$

3. (10pts) Vector $\mathbf{u}$ is perpendicular to the plane containing $\mathbf{w}$ (picture). Their lengths are $||\mathbf{u}|| = 3$ and $||\mathbf{w}|| = 5$. Draw a vector $\mathbf{v}$ whose angle with $\mathbf{u}$ is $\pi/3$ such that $\mathbf{u} \times \mathbf{v} = \mathbf{w}$. What is the length of $\mathbf{v}$?



4. (12pts) Find the points of intersection of the plane $2x - 3y + 4z = 6$ with the x-, y- and z-axes and use this information to sketch the plane in a coordinate system.

5. (20pts) Two lines are given parametrically: $x = -10 - 3t$, $y = 5 + t$, $z = 10 + 2t$ and $x = -5 + 2s$, $y = -2 + 2s$, $z = 6 - s$.
a) Show that these lines intersect by finding the point of intersection.
b) Find the equation of the plane spanned by these two lines.

6. (16pts) This problem is about the surface $\left(\frac{x}{4}\right)^2 - \left(\frac{y}{3}\right)^2 + \left(\frac{z}{2}\right)^2 = 1$.

a) Identify and sketch the intersections of this surface with the coordinate planes.

b) Sketch the surface in 3D, with coordinate system visible.

7. (10pts) Find the cylindrical coordinates of the point whose cartesian coordinates are $(-2\sqrt{3}, 2, 4)$.

8. (10pts) Sketch the following set of points given in spherical coordinates:

$0 \leq \phi \leq \dfrac{\pi}{4}$, $1 \leq \rho \leq 3$

Bonus (10pts) Find a plane that contains the x-axis and has angle $\pi/3$ with the plane $x + y = 4$. How many such planes are there? *(Hints: recall that angle between planes is the angle between their normal vectors. Look for a unit normal vector.)*