

1. (12pts) Simplify and write the answer so all exponents are positive:

a)  $(6x^5y^{-4})^{-2}(2x^{-3}y^4)^5 =$

b)  $\frac{(5w^4z^{-2})^3}{(10w^7z^{-4})^2} =$

2. (4pts) Convert to scientific notation or a decimal number:

$7.7369 \times 10^{-4} =$

$100,873,200 =$

3. (8pts) Simplify and write in standard form:

a)  $(x - 1)(2x + 3) - 3x(x + 1) =$

b)  $(x^2 - 7)(2x^2 - 3x + 4) =$

4. (15pts) Use formulas to expand:

a)  $(2x^2 + 3)^2 =$

b)  $(3x - 5y)(3x + 5y) =$

c)  $(4x - 1)^3 =$

5. (15pts) Factor the following. Use either a known formula or a factoring method.

a)  $x^2 - 7x - 18 =$

b)  $8x^2 - 14x - 15 =$

c)  $8u^3 - v^3 =$

6. (6pts) Verify the formula  $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$  by multiplying out the left side and simplifying.

Algebra & Trigonometry — Joysheet 2  
MAT 150, Fall 2013 — D. Ivanišić

Name: \_\_\_\_\_  
*Show all your work!*

Solve the equations.

1. (3pts)  $7 - 3(2 + x) = 5(x + 3) - 2$

2. (6pts)  $x^2 + x = 2x + 30$

3. (5pts) Solve the equation for  $t$ :

$$at - a = xt + x$$

Simplify.

4. (8pts)  $\frac{4x + 7}{2x^2 + 7x - 30} - \frac{x + 1}{x^2 + 3x - 18} =$

5. (8pts)  $\frac{1 - \frac{7x + 1}{x^2 + 5x - 14}}{3 + \frac{15}{x - 2}} =$

Simplify, showing intermediate steps.

6. (2pts)  $\sqrt{108} =$

7. (4pts)  $\sqrt{175x^3y^{10}} =$

8. (5pts)  $\sqrt[4]{24x^3y}\sqrt[4]{32x^7y^3} =$

9. (8pts) Simplify. Express answers first in terms of positive exponents, then convert to root notation.

$$\frac{\left(27x^{\frac{2}{5}}y^{\frac{3}{2}}\right)^{\frac{4}{3}}}{\left(36x^{-\frac{7}{5}}y^{-\frac{4}{3}}\right)^{\frac{1}{2}}} =$$

10. (5pts) Simplify.

$$(\sqrt{15} + 3\sqrt{3})(2\sqrt{15} - 5\sqrt{3}) =$$

11. (6pts) Rationalize the denominator.

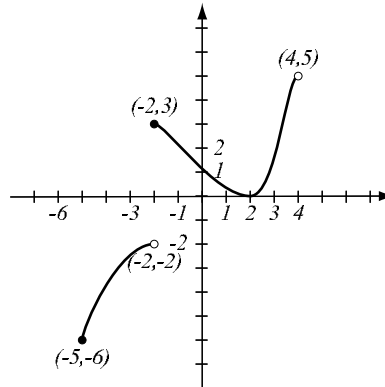
$$\frac{5\sqrt{3} + 2\sqrt{2}}{3\sqrt{2} - 2\sqrt{3}}$$

1. (8pts) Use the distance formula to find out whether the triangle with vertices  $A = (\sqrt{3}, 1)$ ,  $B = (-\sqrt{3}, 1)$  and  $C = (0, -2)$  is an equilateral triangle.

2. (8pts) Write the equation of the circle that contains the point  $A = (-3, 4)$  and whose center is  $C = (-1, -2)$ . Sketch the circle.

3. (8pts) Use the graph of the function  $f$  at right to answer the following questions.

- a) Find  $f(-2)$  and  $f(1)$ .
- b) What is the domain of  $f$ ?
- c) What is the range of  $f$ ?
- d) What are the solutions of the equation  $f(x) = 2$ ?



4. (14pts) The function

$$f(x) = x - 4\sqrt{x+2} + 4 \text{ is given.}$$

a) Use your calculator to accurately draw its graph. Draw the graph here, and indicate units on the axes.

b) Find all the  $x$ - and  $y$ -intercepts (accuracy: 6 decimal points).

c) State the domain and range.

5. (12pts) Find the domain of each function and write it using interval notation.

$$f(x) = \frac{\sqrt[3]{4x-33}}{x^2-5x-14}$$

$$g(x) = \frac{\sqrt[6]{x}}{x^2+21}$$

6. (10pts) Let  $g(x) = \frac{x^2+5x-6}{3x-5}$ . Find the following (simplify where appropriate).

$$g(-2)$$

$$g(5/3)$$

$$g(3a)$$

$$g(x+2)$$



5. (10pts) Millie's Kitchen bought restaurant equipment whose value after three years was estimated at \$12,550. It expects the equipment to last 12 years, at which time they expect to be able to sell it for \$2,200. For tax purposes, they need to know the estimated value  $V(t)$  in every year of operation.

- Write a formula for  $V(t)$ , assuming that it is a linear function (that is, the value decreases by the same amount every year).
- How much did they pay for the equipment when it was new?
- What is the estimated value of the equipment after 9 years?

6. (20pts) A human resources employee tracks the salary of a manager's position in order to model the relationship between time spent with the company and the salary. The table shows the data, where  $S$  is the salary (in thousands), and  $T$  is the number of years in employment with the company.

- Draw the scatterplot of the data. Does the relationship look linear?
- Use two points in the scatterplot to get an equation of a line that models the relationship between  $T$  and  $S$ . Draw the line on the graph.
- Use your calculator to find the "line of best fit" for the data. Draw the line on the graph.
- Find the coefficient of correlation  $r$ . How strong is the linear relationship between  $T$  and  $S$ ?
- What salary can the manager expect after 11 years with the company?

$T$	$S$
1	47.1
2	49.7
4	55.0
6	59.7
7	61.9
8	64.5



**Algebra & Trigonometry — Joysheet 5**  
**MAT 150, Fall 2013 — D. Ivanišić**

**Name:** \_\_\_\_\_  
*Show all your work!*

Solve the inequalities. Write your solution in interval notation.

1. (7pts)  $2x < 4 - 3x < x + 17$  (do as two inequalities)                      2. (7pts)  $3x + 2 \leq 4$  or  $4x - 1 \geq 5$

3. (6pts) Find the domain of the function  $f(x) = \frac{\sqrt{9 - 2x}}{x + 3}$  (in interval notation).

4. (14pts) Michelle is considering two monthly cell-phone plans: plan A costs \$20, includes 100 free minutes and charges 12 cents per minute for additional minutes; plan B costs \$50, includes 400 free minutes and charges 16 cents per minute for additional minutes. Assuming Michelle always uses at least 400 minutes per month, for which number of minutes used is plan B better?

5. (14pts) John B. and Harry R. bike to their jobs on Capitol Hill. It takes John 15 minutes and Harry 20 minutes to ride to work since Harry lives 2 miles farther than John. Harry's bike speed is 4mph more than John's.

a) What are their bike speeds?

b) How far is Capitol Hill from Harry's house?

*(Hint: convert time to hours)*

6. (12pts) How many liters of water must be mixed with 3 liters of a 15% solution of muriatic acid in order to get an 8% solution?

1. (10pts) Use your calculator to accurately sketch the graph of the function  $f(x) = -x^3 + 7x^2 + 6x$ . Draw the graph here, and indicate units on the axes.
- Find the local maxima and minima for this function.
  - State the intervals where the function is increasing and where it is decreasing.

2. (20pts) Let  $f(x) = x^2 + 7x$ ,  $g(x) = \sqrt{3 - x}$ . Find the following (simplify where possible):

$$(f - g)(2) =$$

$$(fg)(-5) =$$

$$(f \circ g)(-1) =$$

$$\frac{f}{g}(x) =$$

$$(f \circ g)(x) =$$

State the domain of  $\frac{g}{f}(x)$

**3.** (8pts) Consider the function  $h(x) = (x^3 + x - 1)^2$ . Find functions  $f$  and  $g$  so that  $h(x) = f(g(x))$ . Find two different solutions to this problem, neither of which is the “stupid” one.

**4.** (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} 2x + 5, & \text{if } -5 < x < -1 \\ 4 - x, & \text{if } -1 \leq x \leq 4. \end{cases}$$

**5.** (14pts) Rancher Diego wishes to enclose an area of  $5\text{km}^2$  in the form of a right triangle so that the length of fence used is minimal.

a) Express the length of the fence used as a function of the length of one of the sides  $x$ . What is the domain of this function?

b) Graph the function in order to find the minimum. What are the dimensions of the enclosure that uses the least fence?

Algebra & Trigonometry — Joysheet 7  
MAT 150, Fall 2013 — D. Ivanišić

Name: \_\_\_\_\_  
*Show all your work!*

Simplify, so that the answer is in form  $a + bi$ .

1. (3pts)  $(3 + 2i)(-4 + i) =$

2. (5pts)  $\frac{1 - i}{3 + 7i} =$

3. (4pts) Simplify and justify your answer.

$i^{995} =$

4. (10pts) Check algebraically whether the graph of  $x^2 + y^2 - 2x = 0$  is symmetric with respect to the  $x$ -axis,  $y$ -axis, or the origin. Then use the calculator to draw the graph and verify your conclusions.

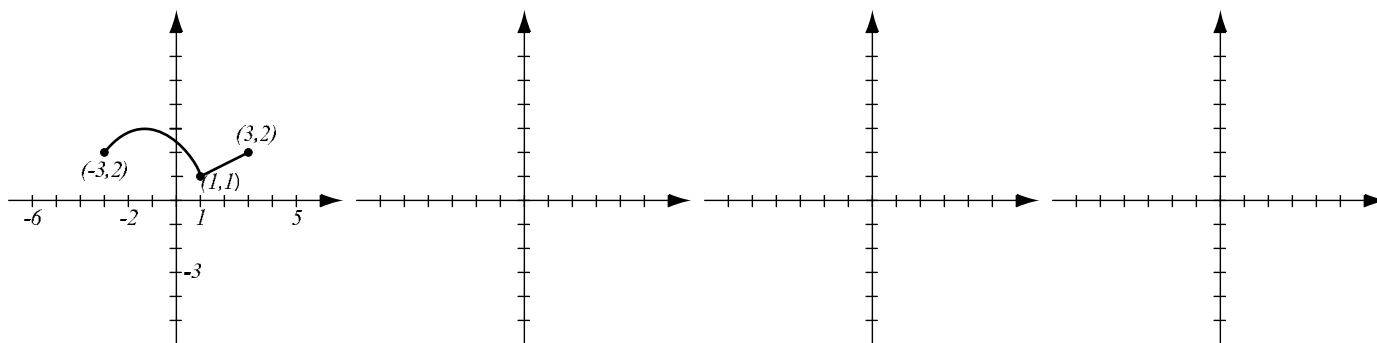
5. (12pts) For the following functions, determine algebraically whether they odd, even, or neither. Then use the calculator to draw their graphs and verify your conclusions.

$f(x) = x^3 - 3x + 7$

$g(x) = x^4 - 5x^2 + 1$

6. (12pts) Using transformations, draw the graphs of  $f(x) = -\frac{1}{x-4}$  and  $g(x) = 2\sqrt{x+4} - 1$ . Explain how you transform graphs of basic functions in order to get the graphs of  $f$  and  $g$ .

7. (14pts) The graph of  $f(x)$  is drawn below. On three separate graphs, sketch the graphs of the functions  $f(x+1)$ ,  $f(-2x)$  and  $2f(x)+1$  and label all the relevant points.



**Algebra & Trigonometry — Joysheet 8**  
**MAT 150, Fall 2013 — D. Ivanišić**

**Name:** \_\_\_\_\_

*Show all your work!*

Solve the equations:

1. (6pts)  $4x^2 + 4 = x^2 + 7x + 1$

2. (8pts)  $x^4 - 3x^2 - 28 = 0$

3. (6pts) Solve by completing the square.

$$x^2 + 14x - 11 = 0$$

4. (12pts) The quadratic function  $f(x) = 2x^2 - 3x - 20$  is given. Do the following without using the calculator.

- Find the  $x$ -intercepts of its graph, if any. Find the  $y$ -intercept.
- Find the vertex of the graph.
- Sketch the graph of the function.

5. (14pts) Rancher Fiona has a rectangular plot of land whose perimeter is 16 miles. If she were to increase the length by 2 miles and width by 3 miles, the area of the land would triple. What are the possible dimensions of the rectangular plot?

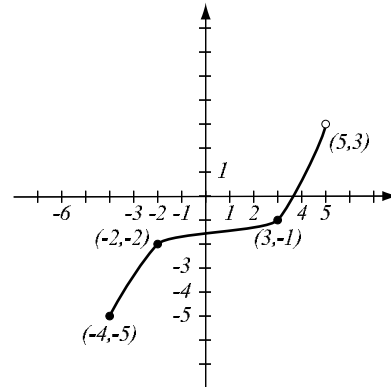
6. (14pts) You have 100 meters of fencing and wish to enclose a rectangular plot that is divided into three pieces by fencing parallel to one of the sides.

a) Express the area of the enclosure as a function of the length of one of the sides. What is the domain of this function?

b) Sketch the graph of the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the enclosure that has the greatest area and what is the greatest area possible?



1. (6pts) The graph of a function  $f$  is given.
- Is this function one-to-one? Justify.
  - If the function is one-to-one, find the graph of  $f^{-1}$ , labeling the relevant points.



2. (12pts) Let  $g(x) = \frac{3x}{2x - 3}$ . Find the formula for  $g^{-1}$ . Find the domain and range of  $g^{-1}$ .

3. (8pts) Evaluate without using the calculator:

$$\log_2 32 = \quad \log_9 \frac{1}{81} = \quad \log_{81} 3 = \quad \log_c \sqrt[6]{c^{11}} =$$

4. (4pts) Use your calculator to find  $\log_5 100$  with accuracy 6 decimal places. Show how you obtained your number.

5. (6pts) If you invest \$4,000 in an account bearing 4.15%, compounded daily, how much is in the account in 3 years?

6. (3pts) Find the domain of  $f(x) = \log_5(3x - 1)$ .

7. (9pts) A restaurant purchased a six-burner range for \$7,000. The value of the range each year is 85% of the value of the preceding year, so its value after  $t$  years is given by  $V(t) = 7000(0.85)^t$ .

a) Sketch the graph of  $V(t)$ .

b) Find the value of the range after 0, 2 and 5 years.

c) Use the intersect feature on the calculator to estimate when the value of the range will be \$1,000

8. (12pts) Using transformations, draw the graphs of  $f(x) = 1 - 2 \cdot 3^x$  and  $g(x) = 2 \log x + 4$ . Explain how you transform graphs of basic functions in order to get the graphs of  $f$  and  $g$ .

1. (6pts) If  $\log_a 3 = 0.380094$  and  $\log_a 7 = 0.673239$ , find (show how you obtained your numbers):

$$\log_a 63 =$$

$$\log_a \frac{49}{3} =$$

$$a =$$

2. (11pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$$\log_4 (64x^2y^5) =$$

$$\log_3 \sqrt[4]{\frac{9x^7y^4}{z^{-2}}} =$$

3. (11pts) Write as a single logarithm. Simplify if possible.

$$2 \log(7x^{\frac{5}{2}}) - \frac{1}{2} \log(196y^8) - \log(x^3) =$$

$$2 \log_a(x^2 - 2x - 8) - 3 \log_a(x - 4) - 2 \log_a(x + 2) =$$

Solve the equations.

4. (5pts)  $3^{2-5x} = \left(\frac{1}{3}\right)^{2x+7}$

5. (7pts)  $7^{x-3} = 3^{1-2x}$

6. (8pts)  $\log_6(x-4) + \log_6(x+1) = 2$

7. (12pts) The number of students enrolled at our fine school increased from 10,025 in 2008 to 10,832 in 2013. Assume the number of students grows exponentially.

a) Write the function describing the number  $P(t)$  of students  $t$  years after 2008. Then find the exponential growth rate of MSU's student population.

b) Graph the function.

c) According to this model, when will Murray State have 14,000 students?

1. (8pts) Without using the calculator, find the exact values of the following trigonometric expressions. Draw the unit circle and the appropriate angle under the expression.

$$\cos 210^\circ = \quad \sec \frac{3\pi}{2} = \quad \tan \left( \frac{7\pi}{4} \right) = \quad \sin(-120^\circ) =$$

2. (4pts) Draw a picture with the unit circle to justify the identity:

$$\sin(\theta + \pi) = -\sin \theta.$$

3. (6pts) Convert to or from radians:

$$165^\circ = \quad \frac{2\pi}{15} =$$

4. (10pts) If  $\sec \theta = \frac{9}{5}$  and  $\theta$  is in the fourth quadrant, find the other five trigonometric functions of  $\theta$ . Draw a picture.

5. (6pts) What distance does the minute hand travel from 5:05PM to 5:23PM if the length of the minute hand is 4 inches?

6. (10pts) A car travels at 80 km/h and has tires with radius 25cm.  
a) What is the angular speed of the wheels?  
b) How many rounds per minute do the wheels turn?

7. (8pts) Using an addition formula and known values of trigonometric functions, find the exact value of  $\cos 165^\circ$ .

8. (8pts) If  $\tan \theta = -\frac{2}{3}$  and  $\theta$  is in the 2nd quadrant, use a double angle formula to find  $\sin 2\theta$ . In which quadrant is  $2\theta$ ?