Algebra & Trigonometry — Joysheet 11
MAT 150, Fall 2013 — D. Ivanšić

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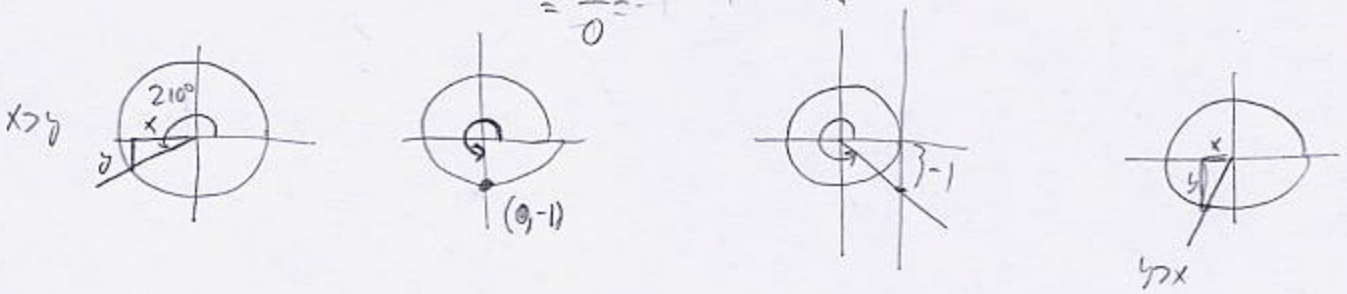
Show all your work!

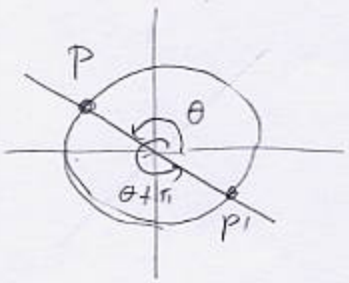
1. (8pts) Without using the calculator, find the exact values of the following trigonometric expressions. Draw the unit circle and the appropriate angle under the expression.

$\cos 210° = -\frac{\sqrt{3}}{2}$ $\sec\frac{3\pi}{2} = \frac{1}{\cos\frac{3\pi}{2}} = \frac{1}{0} =$ undefined $\tan\left(\frac{7\pi}{4}\right) = -1$ $\sin(-120°) = -\frac{\sqrt{3}}{2}$



2. (4pts) Draw a picture with the unit circle to justify the identity:

$\sin(\theta + \pi) = -\sin\theta.$



P and P' have opposite y-coordinates (which are equal to sines)

so $\sin(\theta+\pi) = -\sin\theta$

3. (6pts) Convert to or from radians:

$165° = 165 \cdot \frac{\pi}{180} = \frac{11\pi}{12} = 2.879793$ rad $\quad \frac{2\pi}{15} = \frac{2\pi}{15} \cdot \frac{180°}{\pi} = 24°$

12

1

4. (10pts) If $\sec\theta = \frac{9}{5}$ and θ is in the fourth quadrant, find the other five trigonometric functions of θ. Draw a picture.

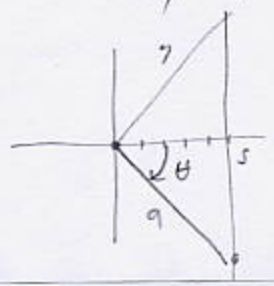
$\sec\theta = \frac{9}{5}$ $\quad 5^2 + y^2 = 9^2$

$\cos\theta = \frac{5}{9} = \frac{x}{r}$ $\quad 25 + y^2 = 81$

$y^2 = 56$

$y = \pm\sqrt{56} =$

$y = -\sqrt{56} = -2\sqrt{14}$ due to Q4



$\sin\theta = -\frac{2\sqrt{14}}{9}$ $\quad \csc\theta = -\frac{9}{2\sqrt{14}}$

$\cos\theta = \frac{5}{9}$ $\quad \sec\theta = \frac{9}{5}$

$\tan\theta = -\frac{2\sqrt{14}}{5}$ $\quad \cot\theta = -\frac{5}{2\sqrt{14}}$

5. (6pts) What distance does the minute hand travel from 5:05PM to 5:23PM if the length of the minute hand is 4 inches?

α is $\frac{18}{60}$ of full circle, $= \frac{3}{10} \cdot 2\pi$ radians $= \frac{3}{5}\pi$ radians

$s = r\theta = 4 \cdot \frac{3}{5}\pi = \frac{12}{5}\pi = 7.539822$ in

6. (10pts) A car travels at 80 km/h and has tires with radius 25cm.
a) What is the angular speed of the wheels?
b) How many rounds per minute do the wheels turn?

a) $v = r\omega$

$80 \text{ km/h} = 25\text{cm} \cdot \omega$

$\frac{80 \text{ km/h}}{25\text{cm}} = \omega$

$\omega = \frac{80 \cdot 1000 \cdot 100 \text{ cm/hr}}{25} =$

$\omega = 320{,}000$ radians/hr

$= 320000/60$ rad/min $= 5333.33$ rad/min

b) $\text{rpm} = \frac{\text{rad/min}}{2\pi} = 848.826363$ rpm

7. (8pts) Using an addition formula and known values of trigonometric funcitons, find the exact value of $\cos 165°$.

$\cos 165° = \cos(120° + 45°) = \cos 120° \cos 45° - \sin 120° \sin 45°$

$= -\frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{-\sqrt{2} - \sqrt{6}}{4} = -\frac{\sqrt{2}+\sqrt{6}}{4}$

$=$

8. (8pts) If $\tan\theta = -\frac{2}{3}$ and θ is in the 2nd quadrant, use a double angle formula to find $\sin 2\theta$. In which quadrant is 2θ?

$\tan\theta = \frac{2}{-3} = \frac{y}{x}$

2nd Quad!

$r = \sqrt{2^2 + (-3)^2} = \sqrt{13}$

$\sin\theta = \frac{2}{\sqrt{13}}$

$\cos\theta = -\frac{3}{\sqrt{13}}$

$\sin 2\theta = 2\sin\theta\cos\theta = 2 \cdot \frac{2}{\sqrt{13}} \cdot \left(-\frac{3}{\sqrt{13}}\right) = -\frac{12}{13}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(-\frac{3}{\sqrt{13}}\right)^2 - \left(\frac{2}{\sqrt{13}}\right)^2 = \frac{9-4}{13} = \frac{5}{13}$

Since $\cos 2\theta > 0$, $\sin 2\theta < 0$

2θ is in quadrant 4.