Algebra & Trigonometry — Joysheet 10	**Name:** Saul Ocean
MAT 150, Fall 2013 — D. Ivanšić	*Show all your work!*

1. (6pts) If $\log_a 3 = 0.380094$ and $\log_a 7 = 0.673239$, find (show how you obtained your numbers):

$\log_a 63 = \log_a (9 \cdot 7)$
$= \log_a (3^2 \cdot 7)$
$= \log_a 3^2 + \log_a 7$
$= 2\log_a 3 + \log_a 7 = 1.433427$

$\log_a \frac{49}{3} = \log_a \frac{7^2}{3}$
$= \log_a 7^2 - \log_a 3$
$= 2\log_a 7 - \log_a 3$
$= 0.966384$

$a =$
$a^{0.380094} = 3 \quad \Big| \, {}^{\wedge}\frac{1}{0.380094}$
$a = 3^{\frac{1}{0.380094}} = 18$

2. (11pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_4 (64x^2y^5) = \log_4 64 + \log_4 x^2 + \log_4 y^5$
$= 3 + 2\log_4 x + 5\log_4 y$
$4^3 = 64$

$\log_3 \sqrt[4]{\frac{9x^7y^4}{z^{-2}}} = \log_3 \left(\frac{9x^7y^4}{z^{-2}}\right)^{\frac{1}{4}} = \frac{1}{4}\log_3\left(\frac{9x^7y^4}{z^{-2}}\right) = \frac{1}{4}\left(\log_3 9 + \log_3 x^7 + \log_3 y^4 - \log_3 z^{-2}\right.$
$= \frac{1}{4}\left(2 + 7\log_3 x + 4\log_3 y + 2\log_3 z\right)$
$= \frac{1}{2} + \frac{7}{4}\log_3 x + \log_3 y + \frac{1}{2}\log_3 z$

3. (11pts) Write as a single logarithm. Simplify if possible.

$2\log(7x^{\frac{5}{2}}) - \frac{1}{2}\log(196y^8) - \log(x^3) = \log (7x^{\frac{5}{2}})^2 - \log(196y^8)^{\frac{1}{2}} - \log x^3$
$= \log 49x^5 - \log(14y^4) - \log x^3$
$= \log \frac{\cancel{49}^{\,7}x^5}{\cancel{14}_{\,2}y^4x^3} = \log \frac{7x^2}{2y^4}$

$2\log_a(x^2 - 2x - 8) - 3\log_a(x-4) - 2\log_a(x+2) =$

$= \log_a (x^2-2x-8)^2 - \log_a (x-4)^3 - \log_a (x+2)^2$
$= \log_a \frac{(x^2-2x-8)^2}{(x-4)^3(x+2)^2} = \log_a \frac{((x-4)(x+2))^2}{(x-4)^3(x+2)^2} = \log_a \frac{\cancel{(x-4)^2}\cancel{(x+2)^2}}{\cancel{(x-4)^3}\cancel{(x+2)^2}} = \log_a \frac{1}{x-4}$

$= \log_a (x-4)^{-1} = -\log_a(x-4)$

Solve the equations.

4. (5pts) $3^{2-5x} = \left(\frac{1}{3}\right)^{2x+7}$

$3^{2-5x} = (3^{-1})^{2x+7}$

$3^{2-5x} = 3^{-2x-7}$

$2-5x = -2x-7 \quad | +2x-2$

$-3x = -9$

$x = 3$

5. (7pts) $7^{x-3} = 3^{1-2x} \quad | \ln$

$\ln 7^{x-3} = \ln 3^{1-2x}$

$(x-3)\ln 7 = (1-2x)\ln 3$

$x\ln 7 - 3\ln 7 = \ln 3 - 2\ln 3 \cdot x \quad \left| \begin{array}{l} +2\ln 3 \cdot x \\ +3\ln 7 \end{array}\right.$

$x\ln 7 + 2\ln 3\, x = \ln 3 + 3\ln 7$

$x(\ln 7 + 2\ln 3) = \ln 3 + 3\ln 7$

$x = \frac{\ln 3 + 3\ln 7}{\ln 7 + 2\ln 3} \approx 1{,}674177$

6. (8pts) $\log_6(x-4) + \log_6(x+1) = 2$

$\log_6((x-4)(x+1)) = 2 \quad | 6^{\,}$

$(x-4)(x+1) = 6^2$

$x^2 - 3x - 4 = 36$

$x^2 - 3x - 40 = 0$

$(x-8)(x+5) = 0$

$x = 8$ or $x = -5$

Check:

$\log_6(8-4) + \log_6(9) \stackrel{?}{=} 2$

$\log_6 4 + \log_6 9 = 2$

$\log_6 36 = 2$ yes

$\log_6(-5-4) + \log_6(-5+1) = 2$

not defined

$\boxed{x=8}$ is only solution

7. (12pts) The number of students enrolled at our fine school increased from 10,025 in 2008 to 10,832 in 2013. Assume the number of students grows exponentially.
a) Write the function describing the number $P(t)$ of students t years after 2008. Then find the exponential growth rate of MSU's student population.
b) Graph the function.
c) According to this model, when will Murray State have 14,000 students?

a) $P(t) = P_0 e^{kt}$ $\quad P_0 = 10{,}025$

t = years since 2008,

$10832 = 10025 e^{k\cdot 5} \quad | \div 10025$

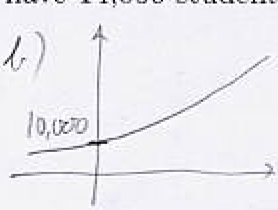
$1{,}0804.. = e^{k\cdot 5} \quad | \ln$

$\ln(1.0804..) = 5k \quad | \div 5$

$k = \frac{\ln(1{,}0804..)}{5} = 0{,}015484845$

about 1.55%

b)



c) $14000 = 10025 e^{kt}$

$\div 10025$

$1{,}39... = e^{kt}$

$\ln 1{,}39.. = kt$

$t = \frac{\ln(1.3965..)}{k} = \frac{\ln(1.3965..)}{0.015484845}$

$= 21{,}568298$ about 21.5 years

Reaches 14,000 in 2029