Algebra and Trigonometry — Joysheet 8
MAT 150, Fall 2013 — D. Ivanšić

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Show all your work!

Solve the equations:

1. (6pts) $4x^2 + 4 = x^2 + 7x + 1$ $\quad | -x^2 - 7x - 1$

$3x^2 - 7x + 3 = 0$

$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4\cdot 3\cdot 3}}{2\cdot 3} = \frac{7 \pm \sqrt{13}}{6}$

2. (8pts) $x^4 - 3x^2 - 28 = 0$

set $u = x^2$

$u^2 - 3u - 28 = 0$

$(u-7)(u+4) = 0$

$u = 7, -4$ $\quad x^2 = 7$ $\quad x^2 = -4$

$x = \pm\sqrt{7}$ $\quad x = \pm 2i$

3. (6pts) Solve by completing the square.

$x^2 + 14x - 11 = 0$ $\quad | + 7^2$

$x^2 + 2x\cdot 7 + 7^2 - 11 = 7^2$

$(x+7)^2 = 49 + 11$

$(x+7)^2 = 60$

$x + 7 = \pm\sqrt{60}$

$x = -7 \pm \sqrt{60}$

$= -7 \pm 2\sqrt{15}$

4. (12pts) The quadratic function $f(x) = 2x^2 - 3x - 20$ is given. Do the following without using the calculator.
a) Find the x-intercepts of its graph, if any. Find the y-intercept.
b) Find the vertex of the graph.
c) Sketch the graph of the function.

a) y-int: $f(0) = -20$

x-int $2x^2 - 3x - 20 = 0$

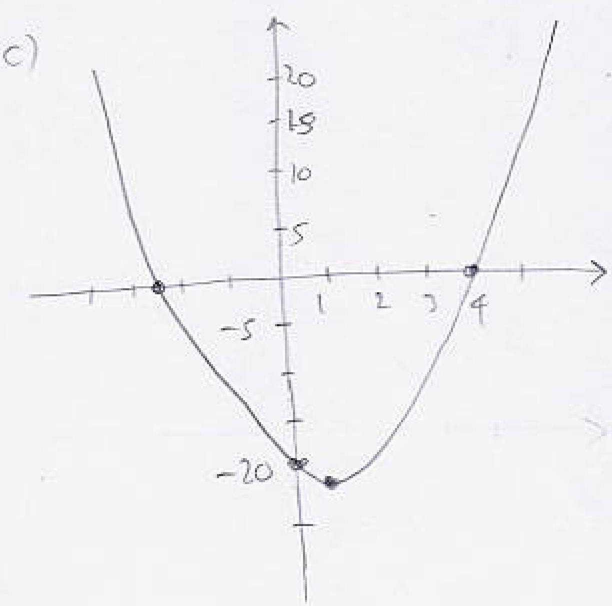
$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4\cdot 2\cdot(-20)}}{2\cdot 2}$

$= \frac{3 \pm \sqrt{169}}{4} = \frac{3 \pm 13}{4} = -\frac{5}{2}, 4$

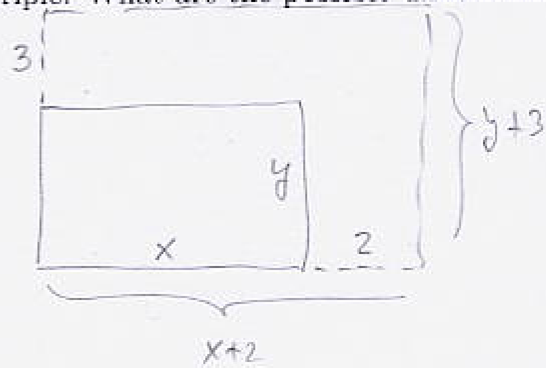
b) $h = -\frac{-3}{2\cdot 2} = \frac{3}{4}$

$k = f\left(\frac{3}{4}\right) = \cancel{2}\cdot\frac{9}{\cancel{16}\,8} - 3\cdot\frac{3}{4} - 20 = \frac{9 - 18 - 160}{8} = -\frac{169}{8} = -21\frac{1}{8}$

c)



5. (14pts) Rancher Fiona has a rectangular plot of land whose perimeter is 16 miles. If she were to increase the length by 2 miles and width by 3 miles, the area of the land would triple. What are the possible dimensions of the rectangular plot?



$(x+2)(y+3)=3xy$

$(x+2)(8-x+3)=3x(8-x)$

$(x+2)(11-x)=24x-3x^2$

$-x^2+9x+22=24x-3x^2 \quad | +3x^2-24x$

$2x^2-15x+22=0$

$x=\frac{-(-15)\pm\sqrt{(-15)^2-4\cdot 2\cdot 22}}{2\cdot 2}=\frac{15\pm\sqrt{49}}{4}=\frac{11}{2}, 2$

$2x+2y=16$

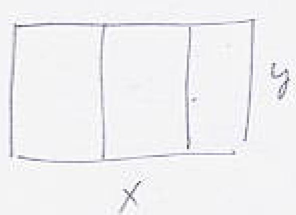
$2y=16-2x \quad | \div 2$

$y=8-x$

Dimensions: 2×6 or $\frac{11}{2}\times\frac{5}{2}$ (5.5×2.5)

6. (14pts) You have 100 meters of fencing and wish to enclose a rectangular plot that is divided into three pieces by fencing parallel to one of the sides.
a) Express the area of the enclosure as a function of the length of one of the sides. What is the domain of this function?
b) Sketch the graph of the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the enclosure that has the greatest area and what is the greatest area possible?

a)



$2x+4y=100$

$2x=100-4y$

$x=50-2y$

$A=xy=(50-2y)y$

$A(y)=-2y^2+50y$

Domain:

$y\geq 0$

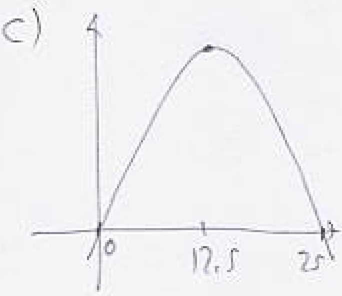
$50-2y\geq 0$

$2y\leq 50$

$y\leq 25$

$[0,25]$

c)



$h=-\frac{50}{2\cdot(-2)}=12.5$

$y=12.5$

$x=50-25=25$

$A(12.5)=(50-25)\cdot 12.5=312.5\ m^2$

Dimensions are 25×12.5