Algebra and Trigonometry — Joysheet 7
MAT 150, Fall 2013 — D. Ivanšić

Name: Saul Ocean

Show all your work!

Simplify, so that the answer is in form $a + bi$.

1. (3pts) $(3+2i)(-4+i) = -12+3i-8i+\underbrace{2i^2}_{=-2} = -14-5i$

2. (5pts) $\dfrac{1-i}{3+7i} = \dfrac{1-i}{3+7i}\cdot\dfrac{3-7i}{3-7i} = \dfrac{3-3i-7i+7i^2}{3^2-(7i)^2} = \dfrac{3-10i-7}{9-(-49)} = \dfrac{-4-10i}{58} = \dfrac{-2-5i}{29}$

$= -\dfrac{2}{29} - \dfrac{5}{29}i$

3. (4pts) Simplify and justify your answer.

$i^{995} = i^{992}\cdot i^3 = \underbrace{(i^4)}_{=1}{}^{248}\cdot i^3 = i^3 = -i$

div. by 4

4. (10pts) Check algebraically whether the graph of $x^2+y^2-2x=0$ is symmetric with respect to the x-axis, y-axis, or the origin. Then use the calculator to draw the graph and verify your conclusions.

x-axis

$(x,y)\to(x,-y)$

$x^2+(-y)^2-2x=0$

x^2+y^2-2x

Same:

y-axis

$(x,y)\to(-x,y)$

$(-x)^2+y^2-2(-x)=0$

$x^2+y^2+2x=0$

not same

origin

$(x,y)\to(-x,-y)$

$(-x)^2+(-y)^2-2(-x)=0$

$x^2+y^2+2x=0$

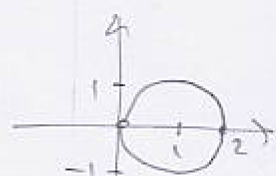
not same

Symm. w.r.t. x-axis only

$x^2+y^2-2x=0$

$y^2=2x-x^2$

$y=\pm\sqrt{2x-x^2}$



5. (12pts) For the following functions, determine algebraically whether they odd, even, or neither. Then use the calculator to draw their graphs and verify your conclusions.

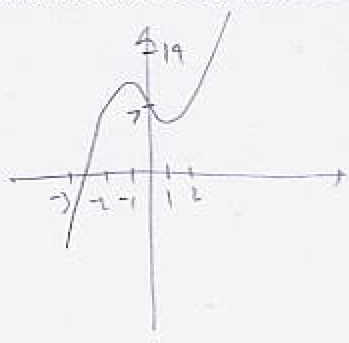
$f(x) = x^3 - 3x + 7$

$f(-x) = (-x)^3 - 3(-x) + 7$

$= -x^3 + 3x + 7$

$\neq f(x), \neq -f(x)$

so neither



Graph has no symmetry

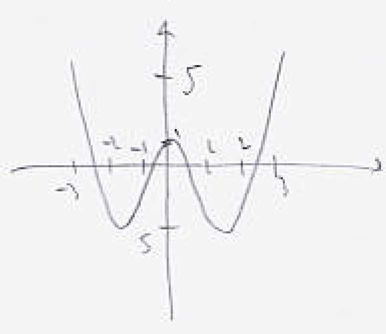
$g(x) = x^4 - 5x^2 + 1$

$f(-x) = (-x)^4 - 5(-x)^2 + 1$

$= x^4 - 5x^2 + 1$

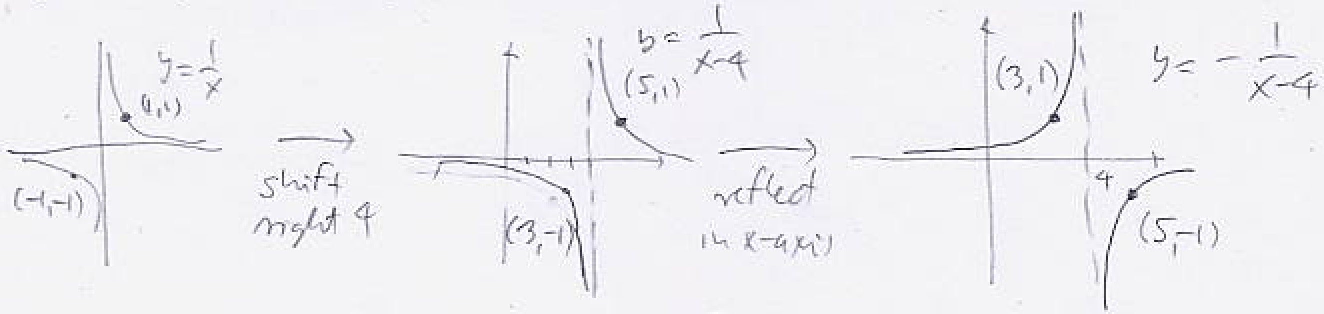
$= f(x)$

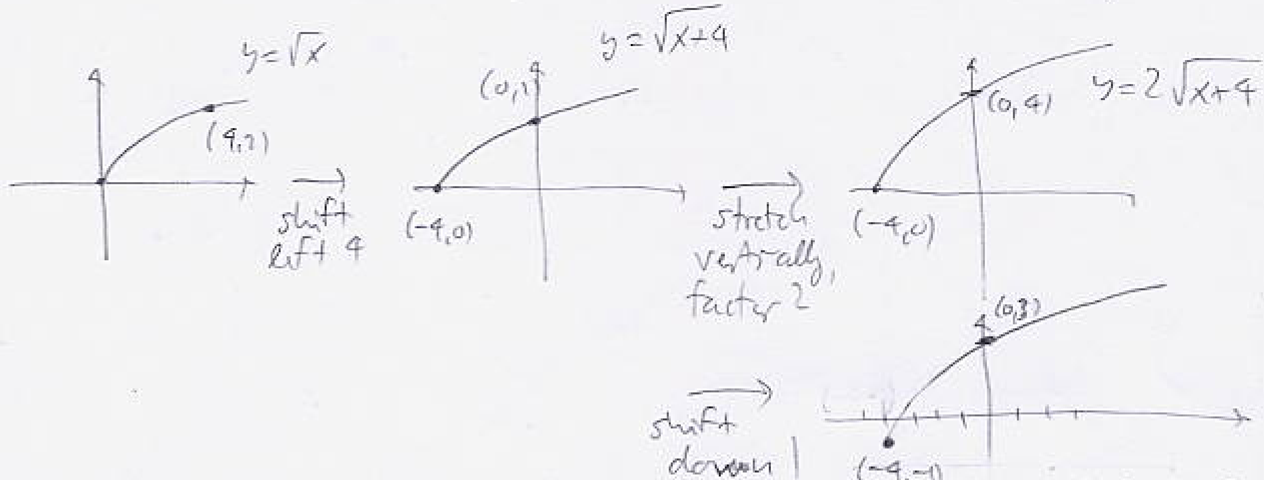
even



Symmetric w.r.t. y-axis

6. (12pts) Using transformations, draw the graphs of $f(x) = -\frac{1}{x-4}$ and $g(x) = 2\sqrt{x+4} - 1$. Explain how you transform graphs of basic functions in order to get the graphs of f and g.





7. (14pts) The graph of $f(x)$ is drawn below. On three separate graphs, sketch the graphs of the functions $f(x+1)$, $f(-2x)$ and $2f(x)+1$ and label all the relevant points.