Algebra and Trigonometry — Joysheet 6 MAT 150, Fall 2013 — D. Ivanšić

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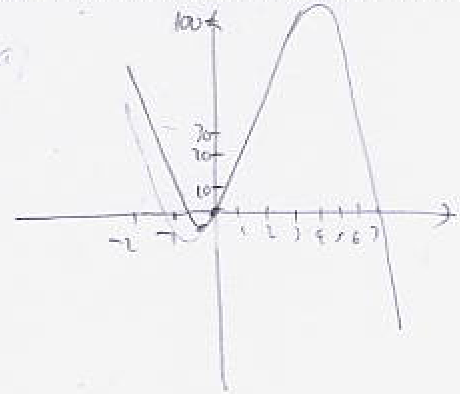
1. (10pts) Use your calculator to accurately sketch the graph of the function $f(x) = -x^3 + 7x^2 + 6x$. Draw the graph here, and indicate units on the axes.
a) Find the local maxima and minima for this function.
b) State the intervals where the function is increasing and where it is decreasing.



a) loc. min -1.216195 at $x = -0.395119$
loc. max 80.03101 at $x = 5.061784$

b) Increasing on $(-0.395119, 5.061784)$
decreasing on $(-\infty, -0.395119) \cup (5.061784, \infty)$

2. (20pts) Let $f(x) = x^2 + 7x$, $g(x) = \sqrt{3-x}$. Find the following (simplify where possible):

$(f-g)(2) = f(2) - g(2)$
$= 4 + 14 - \sqrt{3-2}$
$= 17$

$(fg)(-5) = f(-5) \cdot g(-5)$
$= (25-35)\sqrt{3+5} = -10\sqrt{8}$
$= -20\sqrt{2}$

$(f \circ g)(-1) = f(g(-1))$
$= f(\sqrt{3+1}) = f(2) = 4 + 14 = 18$

$\frac{f}{g}(x) = \frac{f(x)}{g(x)} = \frac{x^2+7x}{\sqrt{3-x}}$

$(f \circ g)(x) = f(g(x)) = f(\sqrt{3-x}) = (\sqrt{3-x})^2 + 7\sqrt{3-x} = 3 - x + 7\sqrt{3-x}$

State the domain of $\frac{g}{f}(x) = \frac{g(x)}{f(x)} = \frac{\sqrt{3-x}}{x^2+7x}$

$x^2 + 7x = 0$
$x(x+7) = 0$
$x = 0, -7$

Domain of g: must have $3 - x \geq 0$
$3 \geq x$
$(-\infty, 3]$

Domain of f: all reals

Domain of $\frac{g}{f}$ = overlap of above with x excluded if $f(x) = 0$

Overlap = $(-\infty, 3]$

$-7 \quad 0 \quad -3$
exclude

Domain = $(-\infty, -7) \cup (-7, 0) \cup (0, -3]$

3. (8pts) Consider the function $h(x) = (x^3 + x - 1)^2$. Find functions f and g so that $h(x) = f(g(x))$. Find two different solutions to this problem, neither of which is the "stupid" one.

$g(x) = x^3 + x - 1$ $\quad$ $g(x) = x^3 + x$

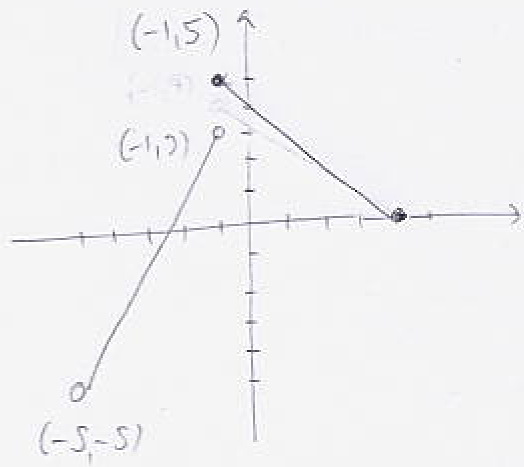
$f(x) = x^2$ $\quad$ $f(x) = (x-1)^2$

4. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} 2x+5, & \text{if } -5 < x < -1 \\ 4-x, & \text{if } -1 \le x \le 4. \end{cases}$$

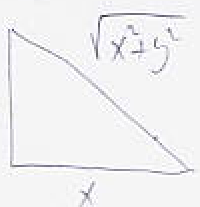
x	$2x+5$
-5	-5
-1	3

x	$4-x$
-1	5
4	0



5. (14pts) Rancher Diego wishes to enclose an area of 5km^2 in the form of a right triangle so that the length of fence used is minimal.
a) Express the length of the fence used as a function of the length of one of the sides x. What is the domain of this function?
b) Graph the function in order to find the minimum. What are the dimensions of the enclosure that uses the least fence?

a)



Area $= 5$

$\frac{1}{2}xy = 5$ $\quad$ 1.2

$xy = 10$

$y = \frac{10}{x}$

$L = x + y + \sqrt{x^2+y^2}$

$= x + \frac{10}{x} + \sqrt{x^2 + \left(\frac{10}{x}\right)^2}$

$L(x) = x + \frac{10}{x} + \sqrt{x^2 + \frac{100}{x^2}}$

Domain: $x \geq 0$ (length)

$x \neq 0$

(or we wouldn't have area $= 5$)

$(0, \infty)$

b)

min occurs for $x = 3.162279$

$y = \frac{10}{3.162279} = 3.162277$

(x and y are equal, so it's an isosceles triangle)