Algebra and Trigonometry — Joysheet 3
MAT 150, Fall 2013 — D. Ivanšić

Name: Saul Ocean

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1. (8pts) Use the distance formula to find out whether the triangle with vertices $A = (\sqrt{3}, 1)$, $B = (-\sqrt{3}, 1)$ and $C = (0, -2)$ is an equilateral triangle.

$d(A,B) = \sqrt{(-\sqrt{3}-\sqrt{3})^2 + (1-1)^2} = \sqrt{(2\sqrt{3})^2} = 2\sqrt{3}$

$d(A,C) = \sqrt{(0-\sqrt{3})^2 + (-2-1)^2} = \sqrt{3+9} = \sqrt{12} = 2\sqrt{3}$

$d(B,C) = \sqrt{(0-(-\sqrt{3}))^2 + (-2-1)^2} = \sqrt{3+9} = \sqrt{12} = 2\sqrt{3}$

since all sides have equal lengths, it is an equilateral triangle

2. (8pts) Write the equation of the circle that contains the point $A = (-3, 4)$ and whose center is $C = (-1, -2)$. Sketch the circle.

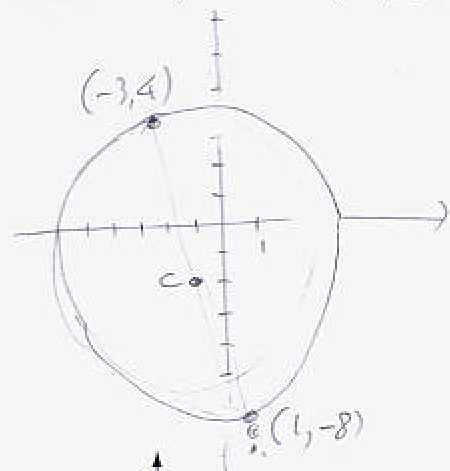
$d(A,C) = \sqrt{(-1-(-3))^2 + (-2-4)^2}$

$= \sqrt{2^2 + (-6)^2}$

$= \sqrt{40}$ = radius

$(x-(-1))^2 + (y-(-2))^2 = \sqrt{40}^2$

$(x+1)^2 + (y+2)^2 = 40$



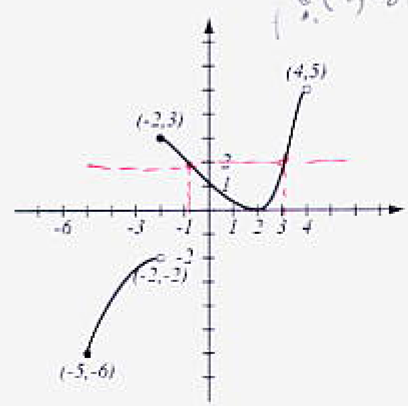
3. (8pts) Use the graph of the function f at right to answer the following questions.

a) Find $f(-2)$ and $f(1)$.

b) What is the domain of f?

c) What is the range of f?

d) What are the solutions of the equation $f(x) = 2$?



a) $f(-2) = 3$

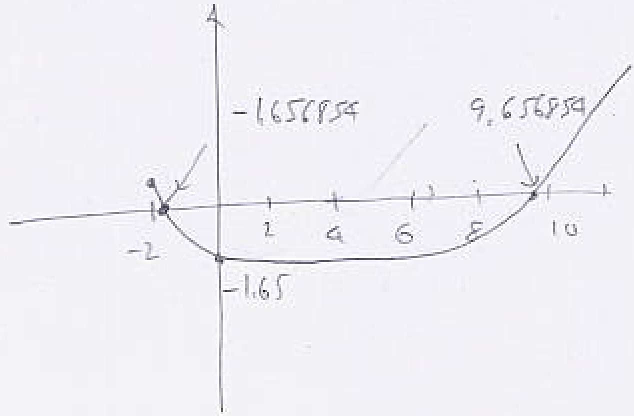
$f(1) = \frac{1}{2}$

b) $[-5, 4]$

c) $[-6, -2) \cup [0, 5]$

d) $x = 3,\ x = -1$

4. (14pts) The function $f(x) = x - 4\sqrt{x+2} + 4$ is given.



a) Use your calculator to accurately its graph. Draw the graph here, and indicate units on the axes.

b) Find all the $x-$ and $y-$intercepts (accuracy: 6 decimal points).

c) State the domain and range.

x-int: -1.656854, 9.656854

y-int: $4 - 4\sqrt{2} \approx -1.656854$

c) domain $= [-2, \infty)$

range $= [-2, \infty)$

5. (12pts) Find the domain of the each function and write it using interval notation.

$$f(x) = \frac{\sqrt[3]{4x-33}}{x^2-5x-14} \qquad\qquad g(x) = \frac{\sqrt[6]{x}}{x^2+21}$$

$4x-33$ can be any number

Can't have $x^2 - 5x - 14 = 0$

$(x-7)(x+2) = 0$

$x = -2, 7$

$\{x \mid x \neq -2 \text{ and } x \neq 7\}$

$(-\infty, -2) \cup (-2, 7) \cup (7, \infty)$

Must have $x \geq 0$

Can't have $x^2 + 21 = 0$

$x^2 = -21$ no solution

$\{x \mid x \geq 0\}$

$[0, \infty)$

6. (10pts) Let $g(x) = \dfrac{x^2+5x-6}{3x-5}$. Find the following (simplify where appropriate).

$g(-2) = \dfrac{(-2)^2 + 5(-2) - 6}{3\cdot(-2) - 5} = \dfrac{-12}{-11} = \dfrac{12}{11}$

$g(5/3) = \dfrac{(\frac{5}{3})^2 + 5\cdot\frac{5}{3} - 6}{3\cdot\frac{5}{3} - 5} = \dfrac{\text{something}}{0}$ not defined

$g(3a) = \dfrac{(3a)^2 + 5\cdot 3a - 6}{3\cdot(3a) - 5} = \dfrac{9a^2 + 15a - 6}{9a - 5}$

$= \dfrac{(9a-3)(a+2)}{9a-5}$

$g(x+2) = \dfrac{(x+2)^2 + 5(x+2) - 6}{3(x+2) - 5}$

$= \dfrac{x^2 + 4x + 4 + 5x + 10 - 6}{3x + 6 - 5}$

$= \dfrac{x^2 + 9x + 8}{3x + 1} = \dfrac{(x+1)(x+8)}{3x+1}$

prod $= -54$ $\quad 18, -3$

sum $= 15$

$9a^2 + 18a - 3a - 6 = 9a(a+2) - 3(a+2)$

$= (9a-3)(a+2)$