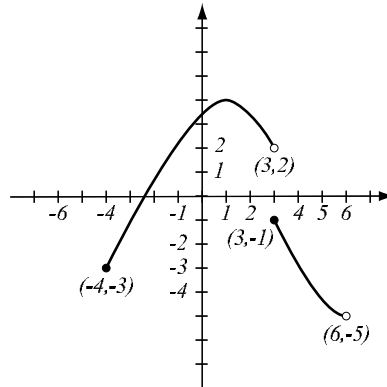


1. (8pts) Use the graph of the function f at right to answer the following questions.

- Find $f(1)$ and $f(3)$.
- What is the domain of f ?
- What is the range of f ?
- What are the solutions of the equation $f(x) = 1$?



2. (10pts) The function $f(x) = x^3 - 6x^2 + 5x - 7$ is given.

- Use your calculator to accurately sketch its graph. Draw the graph here, and indicate units on the axes.
- Find all the x - and y -intercepts (accuracy: 6 decimal points).

3. (4pts) Convert to scientific notation or a decimal number:

$$0.000034612 =$$

$$7.347 \times 10^4 =$$

Use formulas to expand:

4. (4pts) $(2x - 7y)(2x + 7y) =$

5. (4pts) $(x^3 - 2)^2 =$

6. (6pts) $(x - 4)^3 =$

Simplify, showing intermediate steps. Assume all variables are positive.

7. (2pts) $\sqrt{98} =$

8. (5pts) $\sqrt[4]{162x^8y^5} =$

9. (8pts) Express answers first in terms of positive exponents, then convert to root notation.

$$\frac{(16u^2v^{-3})^{\frac{3}{4}}}{3\left(u^{-3}v^{\frac{5}{4}}\right)^2} =$$

10. (8pts) Simplify.

$$\frac{3x + 1}{2x^2 + x - 6} - \frac{x + 3}{x^2 - 3x - 10} =$$

11. (6pts) Rationalize the denominator.

$$\frac{2\sqrt{3} - 5}{\sqrt{3} - 2}$$

12. (8pts) Write the equation of a circle whose diameter has endpoints $A = (3, -2)$ and $B = (-5, 0)$.

13. (4pts) Solve the equation for b .

$$A = \frac{1}{2}h(a + b)$$

14. (7pts) Let $g(x) = x^2 - 3x + 1$. Find the following (simplify where appropriate).

$$g(-1) \qquad g\left(\frac{a}{2}\right)$$

$$g(x - 4)$$

15. (6pts) Find the domain of the function $f(x) = \frac{\sqrt{x}}{x^2 - x - 12}$ and write it in interval notation.

16. (10pts) A business purchases a car for \$26,000. For tax purposes, it is assumed that the car will lose \$3000 in value every year.

a) Write the linear function $V(t)$ that expresses the value of the car after t years.

b) Graph the function $V(t)$ and find $V(5)$.

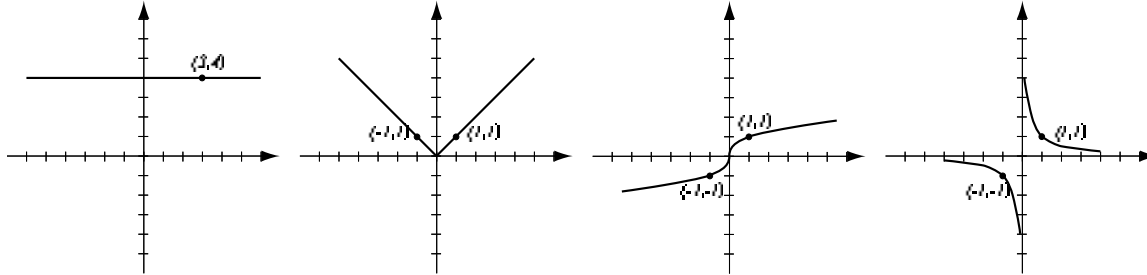
c) What is the meaning of the slope of the line $V(t)$?

Bonus (10pts) Find the equation of the circle that passes through points $(-4, 0)$, $(4, 0)$ and $(0, 2)$. *Hint: due to symmetry, the center must be on the y -axis.*

Algebra & Trigonometry — Exam 2
MAT 150, Fall 2013 — D. Ivanišić

Name: _____
Show all your work!

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



2. (6pts) Simplify, so that the answer is in form $a + bi$.

$$\frac{3 + 5i}{4 - i} =$$

3. (4pts) Simplify and justify your answer.

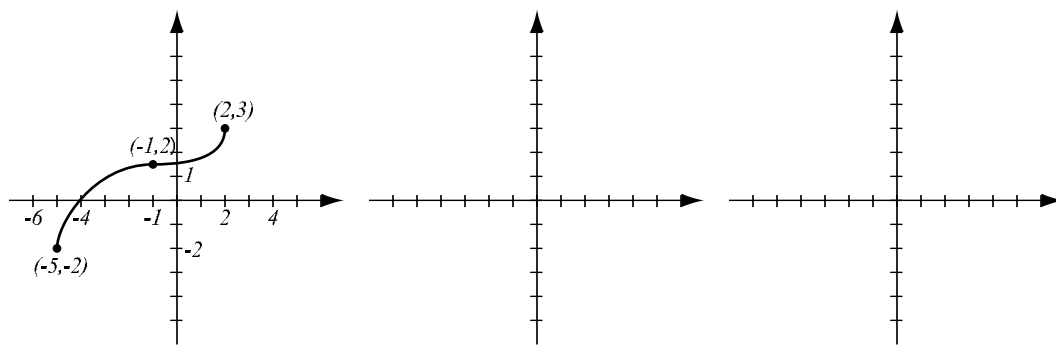
$$i^{77} =$$

4. (8pts) Find the equation of the line (in form $y = mx + b$) that passes through the point $(3, -2)$ and is perpendicular to the line $3x - 5y = 3$. Draw both lines in the coordinate system.

5. (6pts) Solve the inequality. Write your solution in interval notation.

$$-3 \leq 3x + 7 < 12$$

6. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $-f(x+1)$ and $f(2x) - 3$ and label all the relevant points.



7. (13pts) Let $f(x) = \frac{1}{x^2 - 4}$, $g(x) = 3x - 1$.

Find the following (simplify where possible):

$$\frac{f}{g}(x) =$$

$$(f \circ g)(x) =$$

The domain of $\frac{f}{g}$

- 8.** (17pts) Let $f(x) = x^4 - 6x^2 - 8$ (answer with 6 decimal points accuracy).
- Use your graphing calculator to accurately draw the graph of f (on paper!). Indicate scale on the graph.
 - Determine algebraically whether f is even, odd, or neither. Then verify your answer by looking at the graph (justify).
 - Find the local maxima and minima for this function.
 - State the intervals where the function is increasing and where it is decreasing.

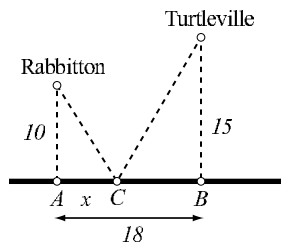
- 9.** (14pts) Gina plans to invest \$12,000, part at 4% and the rest at 6% simple interest. What is the most that she can invest at 4% in order to get at least \$650 per year in interest?

10. (14pts) Fred, who is from Seattle, went to school in Florida. On the way to school, he took a southern route, and on his return after graduation, he took a northern route. On both trips he averaged the same speed. If the southern trek took 52 hours, the northern 60 hours, and the northern trek was 448 miles longer, how long was each trip?

Bonus. (10pts) The towns of Rabbitton and Turtleville are 10 and 15 miles away from the nearest points A and B on an existing highway, which are 18 miles apart. Straight roads from the two towns are planned to join the highway at the same point C so that the total length of the connecting roads is minimized.

a) Express the total length of the connecting roads as a function of x , the distance between A and C . What is the domain of this function?

b) Graph the function in order to find the minimum. Where should the junction C be so that the total length of the connecting roads is the smallest?



Algebra & Trigonometry — Exam 3
MAT 150, Fall 2013 — D. Ivanišić

Name: _____
Show all your work!

Solve the equations:

1. (4pts) $|3 - 2x| - 7 = 5$

2. (6pts) By completing the square: $x^2 + 10x - 3 = 0$

3. (8pts) $\frac{1}{4x + 12} - \frac{1}{x^2 - 9} = \frac{5}{x - 3}$

Solve the inequalities. Draw the solution and write it in interval form.

4. (6pts) $|x - 5| \geq 3$

5. (8pts) $|3x + 4| < 7$

6. (12pts) The quadratic function $f(x) = x^2 - 4x + 7$ is given. Do the following without using the calculator.

a) Find the x - and y -intercepts of its graph, if any.

b) Find the vertex of the graph.

c) Sketch the graph of the function.

Solve the equations:

7. (8pts) $4x^4 + 8x^2 - 5 = 0$

8. (8pts) $2 + \sqrt{2x - 3} = \sqrt{x + 7}$

9. (12pts) A ball is thrown upwards with initial velocity 30 meters per second. The height of the ball in meters t seconds after release is given by the function $s(t) = -5t^2 + 30t$.

a) What is the maximum height that the ball achieves?

b) When does the ball hit the ground?

10. (14pts) The diagonal of a rectangle is $\frac{2}{5}$ the length of its perimeter. If one side of the rectangle has length 1, find the length of the other side .

11. (14pts) Farmer Dwayne has 250 meters of fencing that he will use to enclose a rectangular plot of land next to a straight river. The side of the rectangle along the river does not need fencing. Dwayne wishes to maximize the area of the rectangle.

a) Express the area of the enclosure as a function of the length of one of the sides. What is the domain of this function?

b) Sketch the graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What dimensions of the rectangle give you maximal area?

Bonus. (10pts) Find the domain of the function $f(x) = \sqrt{|x - 7| - |x + 5|}$. (*Hint: use the “distance” interpretation of absolute value.*)

Algebra & Trigonometry — Exam 4
MAT 150, Fall 2013 — D. Ivanišić

Name: _____
Show all your work!

1. (8pts) Evaluate without using the calculator:

$$\log_2 32 = \qquad \log_3 \frac{1}{27} = \qquad \log_a \sqrt[4]{a^5} = \qquad \log_{\sqrt{a}} a^3 =$$

2. (4pts) Use your calculator to find $\log_7 54$ with accuracy 6 decimal places. Show how you obtained your number.

3. (5pts) If $\log_a 6 = 0.588519$ and $\log_a 5 = 0.528634$, find (show how you obtained your numbers):

$$\log_a 30 = \qquad \log_a \frac{5}{36} =$$

4. (6pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$$\log_9(x^2\sqrt{3y^5}) =$$

5. (6pts) Write as a single logarithm. Simplify if possible.

$$2\ln(x-3) - 3\ln(x+4) - 3\ln(x^2+x-12) =$$

6. (9pts) Let $f(x) = 1 - \log(x - 7)$.

a) What is the domain of f ?

b) Explain how you transform the graph of $\log x$ in order to get the graph of f . Indicate at least one point on the graph any asymptotes.

Solve the equations.

7. (8pts) $5^{x^2-7} = 125^{x+1}$

8. (8pts) $5^{x+2} = 2^{x+5}$

9. (12pts) Let $f(x) = x^3 + 6$.

- Draw the graph of f . Is the function one-to-one? Justify.
- Use the graph of f to draw the graph of f^{-1} on the same set of axes.
- Find the formula for f^{-1} .

10. (8pts) If θ is an acute angle and $\cot \theta = \frac{3}{7}$, find the other five trigonometric function values.

11. (4pts) Fill in the blanks using cofunction and reciprocal identities for trigonometric functions.

$$\underline{\hspace{2cm}} 33^\circ = \csc 57^\circ = \frac{1}{\underline{\hspace{2cm}} 57^\circ}$$

12. (10pts) A biologist wishes to estimate the height of a tree. To that end, she moves 40ft away from the trunk and finds that the angle of elevation to the top of the tree is 67° . How tall is the tree?

13. (12pts) According to census data, the population of the Nashville, TN, metro area was about 1,312,000 in 2000 and 1,590,000 in 2010. Assume that the population follows the exponential growth model $P(t) = P_0e^{kt}$.

- a) Write the function that describes the population t years since 2000.
- b) Graph the function on paper.
- c) When will the Nashville metro area reach population 2,000,000?

Bonus (10pts) You take a sighting of the top of a building from a certain point and find that the angle of elevation is 63° . Then you move in a straight line towards the building, pass through the building, exit, and stop some distance away on the other side. Looking back, you see the same top of the building at angle of elevation 72° . If the distance between the points where you took the sightings is 110 meters, how tall is the building?

Algebra & Trigonometry — Exam 5
MAT 150, Fall 2013 — D. Ivanšić

Name: _____
Show all your work!

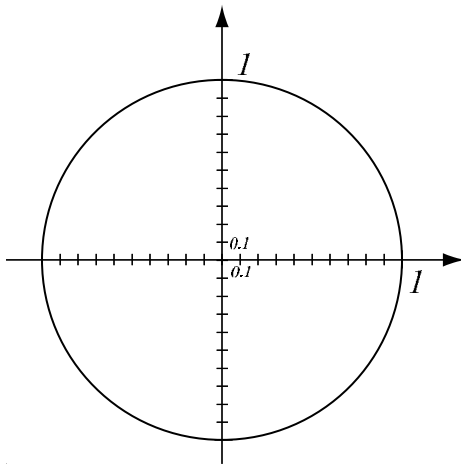
1. (12pts) Without using the calculator, find the exact values of the following trigonometric expressions. Draw the unit circle and the appropriate angle under the expression.

$$\sin 150^\circ = \quad \tan \frac{4\pi}{3} = \quad \cos \left(-\frac{5\pi}{4} \right) = \quad \csc(-300^\circ) =$$

2. (6pts) Convert to or from radians:

$$55^\circ = \quad \frac{3\pi}{20} =$$

3. (10pts) Use the picture below to estimate $\sin \frac{7\pi}{10}$ and $\cos \frac{7\pi}{10}$. Compare your answer with results you get with a calculator.



	estimate	calculator
$\cos \left(\frac{7\pi}{10} \right) =$		
$\sin \left(\frac{7\pi}{10} \right) =$		

4. (10pts) If $\cot \theta = -\frac{3}{2}$ and θ is in the second quadrant, find the other five trigonometric functions of θ . Draw a picture.

5. (12pts) On separate coordinate systems draw accurate picture graphs of $y = \cos \theta$ and $y = \tan \theta$ on the interval $[-2\pi, 2\pi]$. Indicate the x -intercepts.

6. (4pts) Draw a picture with the unit circle to justify the identity $\tan(\theta + \pi) = \tan \theta$.

7. (8pts) A wheel of radius 15 inches rolls on the ground, rotating by angle $\frac{55\pi}{3}$. How far has it traveled?

8. (8pts) A geostationary satellite (of which there are many) orbits Earth so that it is always above the same spot on the Earth. In order to achieve this, it has to rotate with the same angular velocity as the Earth at distance 42,164 kilometers from Earth's center. What is a geostationary satellite's linear speed in kilometers per hour?

Use trigonometric identities to simplify:

9. (6pts) $(\tan \theta + \cot \theta) \sin \theta \cos \theta =$

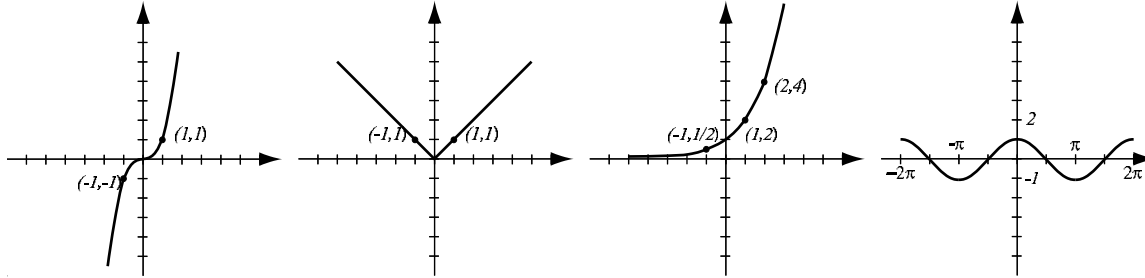
10. (6pts) $\sin\left(\frac{\pi}{2} - \theta\right) \sec \theta - \cos\left(\frac{\pi}{2} - \theta\right) \csc \theta =$

11. (8pts) Using an addition formula and known values of trigonometric functions, find the exact value of $\sin 105^\circ$.

12. (10pts) If $\cos \theta = -\frac{2}{7}$ and θ is in the 3rd quadrant, use a double angle formula to find $\sin 2\theta$ and $\cos 2\theta$. In which quadrant is 2θ ?

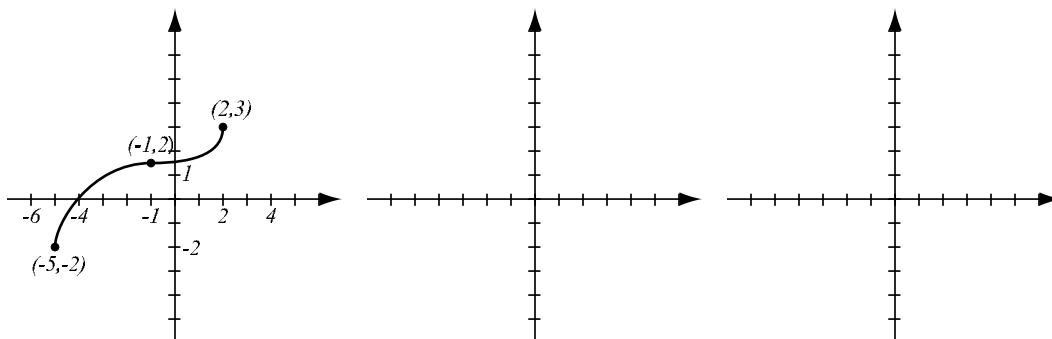
Bonus (10pts) Use double-angle formulas to find $\cos \frac{\pi}{8}$ and $\sin \frac{\pi}{8}$. (*Hint: put $\theta = \frac{\pi}{8}$ into a double-angle formula and solve for the quantity that you don't know.*)

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



2. (8pts) Find the equation of the line (in form $y = mx + b$) that passes through the points $(1, -2)$ and $(3, 1)$. Then write the equation of the line perpendicular to it that passes through $(1, -2)$. Draw both lines in the coordinate system.

3. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $f(-x) + 3$ and $3f(x - 2)$ and label all the relevant points.



4. (6pts) Find the domain of the function $f(x) = \frac{\sqrt{7-x}}{x^2 - 11x + 18}$ and write it in interval notation.

5. (6pts) Solve the inequality. Draw the solution and write it in interval notation.

$$|x - 4| \geq 1$$

6. (20pts) Let $f(x) = x^3 - 24x$ (answer with 6 decimal points accuracy).

a) Use your graphing calculator to accurately draw the graph of f (on paper!). Indicate scale on the graph.

b) Determine algebraically whether f is even, odd, or neither. Then verify your answer by looking at the graph (justify).

c) Find the local maxima and minima for this function.

d) State the intervals where the function is increasing and where it is decreasing.

e) How many solutions does the equation $f(x) = 63$ have? (You do not have to find the solutions.)

7. (14pts) The quadratic function $f(x) = 2x^2 - x - 15$ is given. Do the following without using the calculator.

- Find the x - and y -intercepts of its graph, if any.
- Find the vertex of the graph.
- Sketch the graph of the function.
- What is the range of f ?

8. (6pts) If $\log_a 3 = 0.458157$ and $\log_a 5 = 0.671188$, find (show how you obtained your numbers):

$$\log_a 15 =$$

$$\log_a \frac{9}{125} =$$

9. (10pts) Without using the calculator, find the exact values of the following trigonometric expressions. Draw the unit circle and the appropriate angle under the expression.

$$\cos 135^\circ =$$

$$\cot \frac{5\pi}{3} =$$

$$\sin \left(-\frac{2\pi}{3} \right) =$$

$$\csc(-60^\circ) =$$

10. (6pts) A 30-ft ladder leans against the wall and makes an angle of 79° with the ground. How high up the wall does the ladder reach?

11. (10pts) If $\csc \theta = -\frac{5}{3}$ and θ is in the third quadrant, find the other five trigonometric functions of θ . Draw a picture.

12. (6pts) What distance does the tip of the minute hand travel from 4:00PM to 5:24PM if the length of the minute hand is 2 inches?

13. (12pts) According to census data, the population of the Nashville, TN, metro area was about 1,312,000 in 2000 and 1,590,000 in 2010. Assume that the population follows the exponential growth model $P(t) = P_0e^{kt}$.

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a) Express the area of the enclosure as a function of the length of one of the sides. What is the domain of this function?

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Bonus (10pts) Find the equation of the circle that passes through points $(-4, 0)$, $(4, 0)$ and $(0, 2)$. *Hint: due to symmetry, the center must be on the y -axis.*