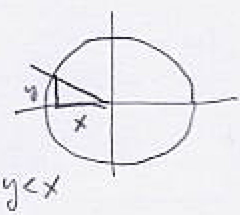
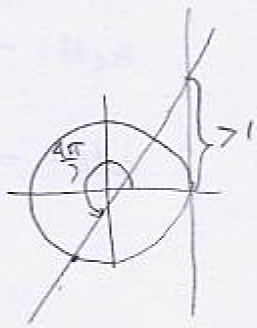
Algebra and Trigonometry — Exam 5 **MAT 150, Fall 2013 — D. Ivanšić**	**Name:** Saul Ocean *Show all your work!*

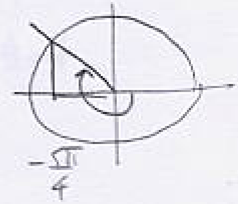
1. (12pts) Without using the calculator, find the exact values of the following trigonometric expressions. Draw the unit circle and the appropriate angle under the expression.

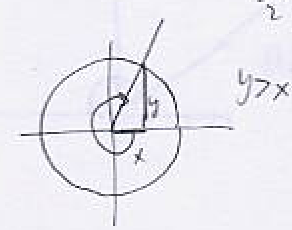
$\sin 150° = \frac{1}{2}$ $\qquad \tan\frac{4\pi}{3} = \sqrt{3}$ $\qquad \cos\left(-\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2}$ $\qquad \csc(-300°) = \frac{1}{\sin(-300°)}$

$= \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$





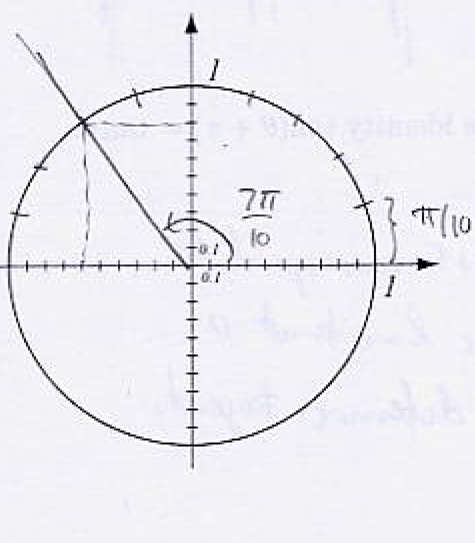




2. (6pts) Convert to or from radians:

$55° = 55 \cdot \frac{\pi}{180°} = \frac{11}{36}\pi = 0.959931$ $\qquad \frac{3\pi}{20} = \frac{3\cancel{\pi}}{\cancel{20}_{1}} \cdot \frac{\cancel{180}^{9}°}{\cancel{\pi}} = 27°$

3. (10pts) Use the picture below to estimate $\sin\frac{7\pi}{10}$ and $\cos\frac{7\pi}{10}$. Compare your answer with results you get with a calculator.

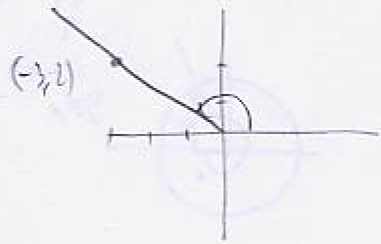


	estimate	calculator
$\cos\left(\frac{7\pi}{10}\right) =$	-0.6	-0.58
$\sin\left(\frac{7\pi}{10}\right) =$	0.8	0.81

4. (10pts) If $\cot\theta = -\frac{3}{2}$ and θ is in the second quadrant, find the other five trigonometric functions of θ. Draw a picture.

$\cot\theta = -\frac{3}{2}$ $\qquad r = \sqrt{2^2+(-3)^2} = \sqrt{13}$

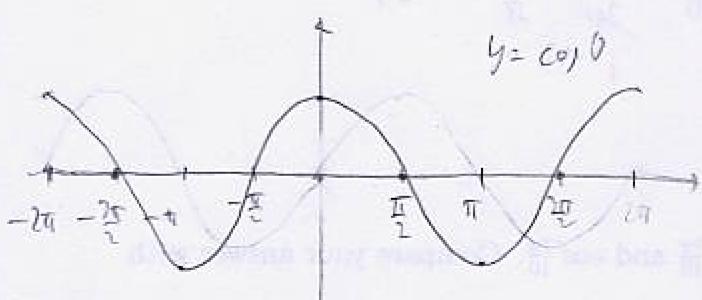
$\tan\theta = -\frac{2}{3} = \frac{y}{x} = \frac{2}{-3}$ $\qquad \sin\theta = \frac{2}{\sqrt{13}}$ $\qquad \csc\theta = \frac{\sqrt{13}}{2}$

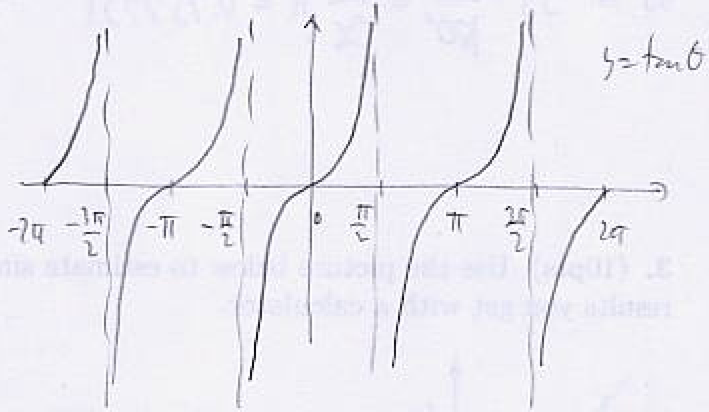


$\cos\theta = \frac{-3}{\sqrt{13}}$ $\qquad \sec\theta = -\frac{\sqrt{13}}{3}$

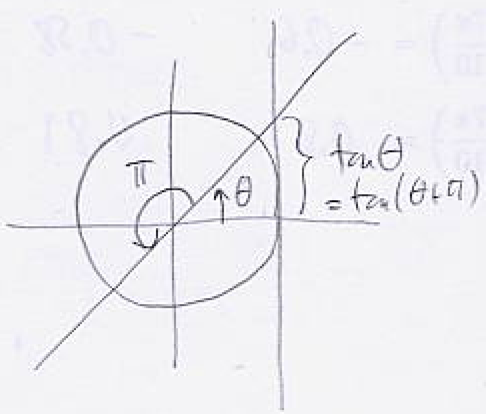
$\tan\theta = -\frac{2}{3}$ $\qquad \cot\theta = -\frac{3}{2}$

5. (12pts) On separate coordinate systems draw accurate picture graphs of $y = \cos\theta$ and $y = \tan\theta$ on the interval $[-2\pi, 2\pi]$. Indicate the x-intercepts.



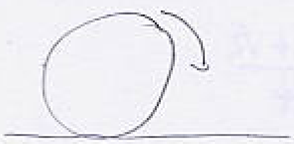


6. (4pts) Draw a picture with the unit circle to justify the identity $\tan(\theta + \pi) = \tan\theta$.



θ and θ+π correspond to the same line that is used to determine tangent.

7. (8pts) A wheel of radius 15 inches rolls on the ground, rotating by angle $\frac{55\pi}{3}$. How far has it traveled?



$s = r\theta = \overset{5}{\cancel{15}} \cdot \frac{55\pi}{\underset{1}{\cancel{3}}} = 275\pi = 863.937980$ in

8. (8pts) A geostationary satellite (of which there are many) orbits Earth so that it is always above the same spot on the Earth. In order to achieve this, it has to rotate with the same angular velocity as the Earth at distance 42,164 km from Earth's center. What is a geostationary satellite's linear speed in kilometers per hour?

$v = r\omega$

$= 42164 \cdot \frac{2\pi}{24}$ km/h $= \frac{10541\pi}{3} = 11{,}038.50939$ km/h

1 rev in 24 hrs $= \frac{2\pi \text{ rad}}{24 \text{ hrs}}$

About 11,000 km/hr

Use trigonometric identities to simplify:

9. (6pts) $(\tan\theta + \cot\theta)\sin\theta\cos\theta =$ $\tan\theta\sin\theta\cos\theta + \cot\theta\sin\theta\cos\theta$

$= \frac{\sin\theta}{\cancel{\cos\theta}}\sin\theta\cancel{\cos\theta} + \frac{\cos\theta}{\cancel{\sin\theta}}\cdot\cancel{\sin\theta}\cos\theta$

$= \sin^2\theta + \cos^2\theta = 1$

10. (6pts) $\sin\left(\frac{\pi}{2}-\theta\right)\sec\theta - \cos\left(\frac{\pi}{2}-\theta\right)\csc\theta =$ $\cos\theta\cdot\sec\theta - \sin\theta\csc\theta$

cofunction identities

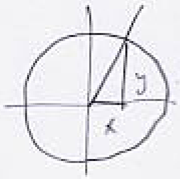
$= \cos\theta\frac{1}{\cos\theta} - \sin\theta\cdot\frac{1}{\sin\theta}$

$= 1 - 1 = 0$

11. (8pts) Using an addition formula and known values of trigonometric funcitons, find the exact value of $\sin 105°$.

$\sin 105° = \sin(60°+45°) = \sin 60° \cos 45° + \cos 60° \sin 45°$

$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}$

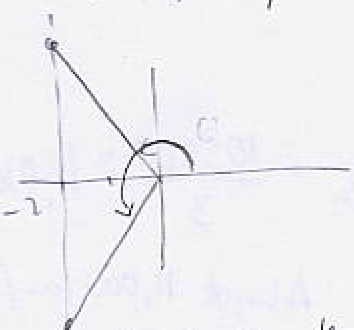


$x < y$

12. (10pts) If $\cos\theta = -\frac{2}{7}$ and θ is in the 3rd quadrant, use a double angle formula to find $\sin 2\theta$ and $\cos 2\theta$. In which quadrant is 2θ?

$\cos\theta = \frac{x}{r} = \frac{-2}{7}$

$\sin 2\theta = 2\sin\theta\cos\theta = 2 \cdot \frac{-3\sqrt{5}}{7} \cdot \left(\frac{-2}{7}\right) = \frac{12\sqrt{5}}{49}$



$\sin\theta = \frac{3\sqrt{5}}{7}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(-\frac{2}{7}\right)^2 - \left(\frac{3\sqrt{5}}{7}\right)^2$

$= \frac{4}{49} - \frac{45}{49} = -\frac{41}{49}$

$\left(\begin{array}{l} y>0 \\ x<0 \end{array} \right.$ II

Since $\sin 2\theta > 0$, $\cos 2\theta < 0$

2θ is in Quadrant 2

$(-2)^2 + y^2 = 7^2$ $\quad y = \pm\sqrt{45}$

$4 + y^2 = 49$ $\quad y = -\sqrt{45} = -3\sqrt{5}$

$y^2 = 45$ $\quad$ (bt in Q3)

Bonus (10pts) Use double-angle formulas to find $\cos\frac{\pi}{8}$ and $\sin\frac{\pi}{8}$. *(Hint: put $\theta = \frac{\pi}{8}$ into a double-angle formula and solve for the quantity that you don't know.)*

$\sin 2\theta = 2\sin\theta\cos\theta$

$\sin 2 \cdot \frac{\pi}{8} = 2\sin\frac{\pi}{8}\cos\frac{\pi}{8}$

$\sin\frac{\pi}{4} = 2\sin\frac{\pi}{8}\cos\frac{\pi}{8}$

$\frac{\sqrt{2}}{2} = \cancel{2}\sin\frac{\pi}{8} \cdot \frac{\sqrt{2+\sqrt{2}}}{\cancel{2}}$

$\sin\frac{\pi}{8} = \frac{\sqrt{2}}{2\sqrt{2+\sqrt{2}}}$

$\cos 2\theta = 2\cos^2\theta - 1$

$\cos 2 \cdot \frac{\pi}{8} = 2\cos^2\frac{\pi}{8} - 1$

$\cos\frac{\pi}{4} = 2\cos^2\frac{\pi}{8} - 1$

$\frac{\sqrt{2}}{2} = 2\cos^2\frac{\pi}{8} - 1$

$2\cos^2\frac{\pi}{8} = 1 + \frac{\sqrt{2}}{2}$

$\cos^2\frac{\pi}{8} = \frac{1+\frac{\sqrt{2}}{2}}{2} = \frac{2+\sqrt{2}}{4}$

$\cos\frac{\pi}{8} = \pm\sqrt{\frac{2+\sqrt{2}}{4}}$

$\cos\frac{\pi}{8} = \frac{\sqrt{2+\sqrt{2}}}{2}$ $\quad$ (+ since $\frac{\pi}{8}$ is in Q1)