Algebra and Trigonometry — Exam 4
MAT 150, Fall 2013 — D. Ivanšić

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Show all your work!

1. (8pts) Evaluate without using the calculator:

$\log_2 32 = 5$ $\quad$ $\log_3 \frac{1}{27} = -3$ $\quad$ $\log_a \sqrt[4]{a^5} = \frac{5}{4}$ $\quad$ $\log_{\sqrt{a}} a^3 = 6$

$2^? = 32$ $\quad$ $3^? = \frac{1}{27} = \frac{1}{3^3}$ $\quad$ $a^? = \sqrt[4]{a^5} = a^{\frac{5}{4}}$ $\quad$ $\sqrt{a}^? = a^3$ $\quad$ $\sqrt{a} = a^{\frac{1}{2}}$ $\left(a^{\frac{1}{2}}\right)^6 = a^3$

2. (4pts) Use your calculator to find $\log_7 54$ with accuracy 6 decimal places. Show how you obtained your number.

$\log_7 54 = \frac{\ln 54}{\ln 7} = 2.049932$

change of base formula

3. (5pts) If $\log_a 6 = 0.588519$ and $\log_a 5 = 0.528634$, find (show how you obtained your numbers):

$\log_a 30 = \log_a (5 \cdot 6)$
$= \log_a 5 + \log_a 6 = 0.528634 + 0.588519$
$= 1.117153$

$\log_a \frac{5}{36} = \log_a 5 - \log_a 6^2$
$= \log_a 5 - 2\log_a 6 = 0.528634 - 2 \cdot 0.588519$
$= -0.648404$

4. (6pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_9(x^2\sqrt{3y^5}) = \log_9 x^2 + \log_9 (3y^5)^{\frac{1}{2}} = 2\log_9 x + \frac{1}{2}\log_9(3y^5)$

$= 2\log_9 x + \frac{1}{2}\left(\log_9 3 + \log_9 y^5\right) = 2\log_9 x + \frac{1}{2}\left(\frac{1}{2} + 5\log_9 y\right)$

$= 2\log_9 x + \frac{1}{4} + \frac{5}{2}\log_9 y$

5. (6pts) Write as a single logarithm. Simplify if possible.

$2\ln(x-3) - 3\ln(x+4) - 3\ln(x^2+x-12) = \ln(x-3)^2 - \ln(x+4)^3 - \ln(\underbrace{x^2+x-12}_{(x-3)(x+4)})^3$

$= \ln \frac{(x-3)^2}{(x+4)^3\left((x-3)(x+4)\right)^3} = \ln \frac{(x-3)^2}{(x+4)^3(x-3)^3(x+4)^3}$

$= \ln \frac{1}{(x+4)^6(x-3)}$

6. (9pts) Let $f(x) = 1 - \log(x-7)$. $= -\log(x-7)+1$

a) What is the domain of f?

b) Explain how you transform the graph of $\log x$ in order to get the graph of f. Indicate at least one point on the graph any asymptotes.

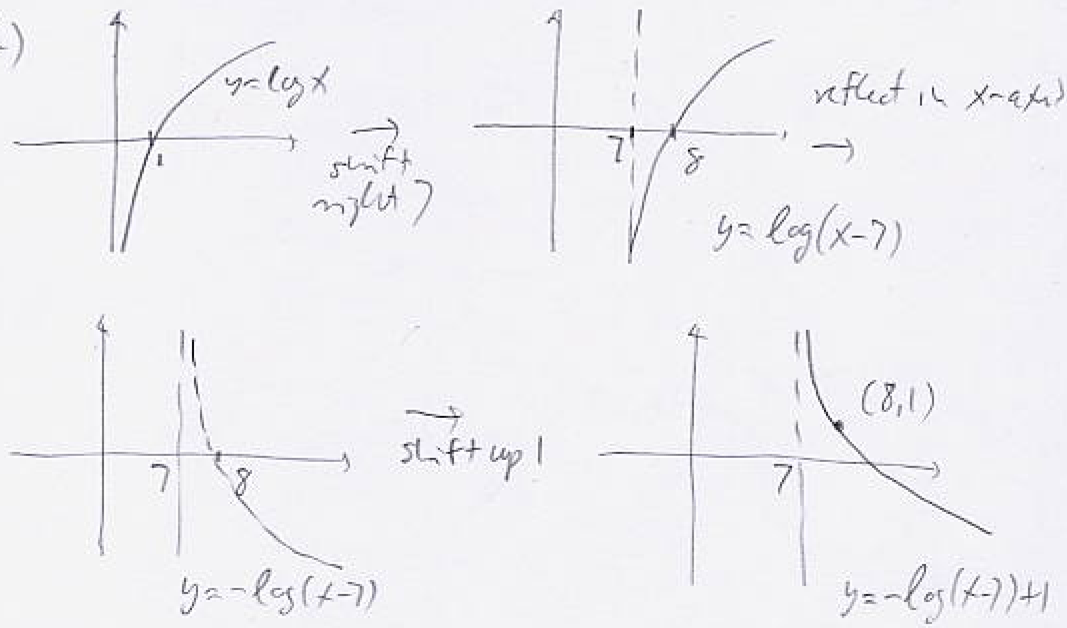
a) Must have

$x-7>0$

$x>7$

$(7,\infty)$

b)



Solve the equations.

7. (8pts) $5^{x^2-7} = 125^{x+1}$

$5^{x^2-7} = (5^3)^{x+1}$

$5^{x^2-7} = 5^{3x+3}$

$x^2-7=3x+3$

$x^2-3x-10=0$

$(x-5)(x+2)=0$

$x=5,-2$

8. (8pts) $5^{x+2} = 2^{x+5}$ $\mid \ln$

$\ln 5^{x+2} = \ln 2^{x+5}$

$(x+2)\ln 5 = (x+5)\ln 2$

$x\ln 5 + 2\ln 5 = x\ln 2 + 5\ln 2 \quad \mid -x\ln 2 - 2\ln 5$

$x\ln 5 - x\ln 2 = 5\ln 2 - 2\ln 5$

$x(\ln 5 - \ln 2) = 5\ln 2 - 2\ln 5$

$x = \dfrac{5\ln 2 - 2\ln 5}{\ln 5 - \ln 2}$

$= 0.269412$

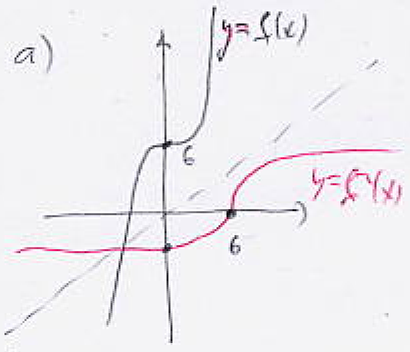
9. (12pts) Let $f(x) = x^3 + 6$.

a) Draw the graph of f. Is the function one-to-one? Justify.

b) Use the graph of f to draw the graph of f^{-1} on the same set of axes.

c) Find the formula for f^{-1}.

a)



It passes the horizontal line test, so is one-to-one

$y = x^3 + 6$ is $y = x^3$ shifted up 6

c) $y = x^3 + 6$

$y - 6 = x^3$

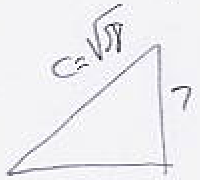
$x = \sqrt[3]{y-6}$

$f^{-1}(y) = \sqrt[3]{y-6}$

or $f^{-1}(x) = \sqrt[3]{x-6}$

10. (8pts) If θ is an acute angle and $\cot\theta = \frac{3}{7}$, find the other five trigonometric function values.

$\cot\theta = \frac{3}{7}$ so $\tan\theta = \frac{7}{3} = \frac{opp}{adj}$



$c = \sqrt{58}$

$c^2 = 3^2 + 7^2 = 58$

$\sin\theta = \frac{7}{\sqrt{58}}$ $\quad \csc\theta = \frac{\sqrt{58}}{7}$

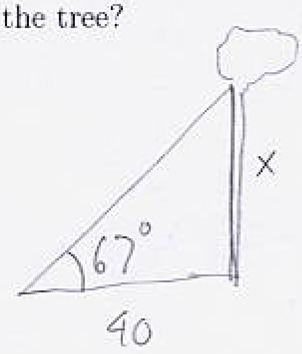
$\cos\theta = \frac{3}{\sqrt{58}}$ $\quad \sec\theta = \frac{\sqrt{58}}{3}$

$\tan\theta = \frac{7}{3}$ $\quad \cot\theta = \frac{3}{7}$

11. (4pts) Fill in the blanks using cofunction and reciprocal identities for trigonometric functions.

$\underline{\sec}\ 33° = \csc 57° = \dfrac{1}{\underline{\sin}\ 57°}$

12. (10pts) A biologist wishes to estimate the height of a tree. To that end, she moves 40ft away from the trunk and finds that the angle of elevation to the top of the tree is 67°. How tall is the tree?



$\frac{x}{40} = \tan 67°$

$x = 40 \tan 67°$

$= 94.234095$ ft

13. (12pts) According to census data, the population of the Nashville, TN, metro area was about 1,312,000 in 2000 and 1,590,000 in 2010. Assume that the population follows the exponential growth model $P(t) = P_0 e^{kt}$.

a) Write the function that describes the population t years since 2000.

b) Graph the function on paper.

c) When will the Nashville metro area reach population 2,000,000?

a) P(t) in thousands

$$P(t) = 1312\, e^{kt}$$

$$1590 = 1312\, e^{k \cdot 10}$$

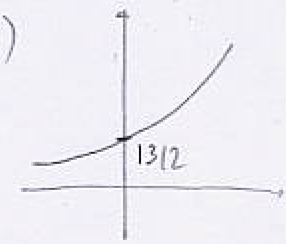
$$\frac{1590}{1312} = e^{k\cdot 10} \quad | \ln$$

$$\ln \frac{1590}{1312} = k \cdot 10$$

$$k = \frac{1}{10} \ln \frac{1590}{1312} = 0.0192181$$

About 1.9% growth rate

b)



c)

$$2000 = 1312\, e^{0.019\ldots t}$$

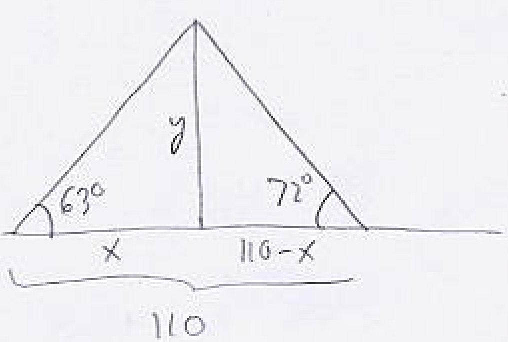
$$\frac{2000}{1312} = e^{0.019\ldots t} \quad | \ln$$

$$\ln \frac{2000}{1312} = 0.019\ldots t$$

$$t = \frac{\ln \frac{2000}{1312}}{0.0192181} = 21.937329$$

Population will be 2,000,000 about 22 years from 2000, so in 2022.

Bonus (10pts) You take a sighting of the top of a building from a certain point and find that the angle of elevation is 63°. Then you move in a straight line towards the building, pass through the building, exit, and stop some distance away on the other side. Looking back, you see the same top of the building at angle of elevation 72°. If the distance between the points where you took the sightings is 110 meters, how tall is the building?



$$\begin{cases} \frac{y}{x} = \tan 63° \\ \frac{y}{110-x} = \tan 72° \end{cases} \qquad \begin{cases} \frac{x}{y} = \cot 63° \\ \frac{110-x}{y} = \cot 72° \end{cases} \qquad x = y\cot 63°$$

so

$$110 - y\cot 63° = y \cot 72°$$

$$y \cot 72° + y \cot 63° = 110$$

$$y(\cot 72° + \cot 63°) = 110$$

$$y = \frac{110}{\cot 72° + \cot 63°} = 131.824124 \text{ meters}$$ height of building

$$(x = y \cot 63° = 67.167746)$$