Algebra and Trigonometry — Exam 3
MAT 150, Fall 2013 — D. Ivanšić

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Show all your work!

Solve the equations:

1. (4pts) $|3-2x|-7=5$

$|3-2x|=12$

$3-2x=12$ or $3-2x=-12$

$-2x=9 \qquad -2x=-15$

$x=-\frac{9}{2} \qquad x=\frac{15}{2}$

2. (6pts) By completing the square: $x^2+10x-3=0 \quad | +5^2$

$\underbrace{x^2+2\cdot x\cdot 5+5^2}-3=5^2$

$(x+5)^2=25+3$

$(x+5)^2=28$

$x+5=\pm\sqrt{28}$

$x=-5\pm\sqrt{28}$

$=-5\pm 2\sqrt{7}$

3. (8pts) $\dfrac{1}{4x+12}-\dfrac{1}{x^2-9}=\dfrac{5}{x-3} \quad | \cdot 4(x+3)(x-3)$

$4(x+3) \quad (x-3)(x+3) \quad x-3$

$x-3-4=5\cdot 4(x+3)$

$x-7=20x+60$

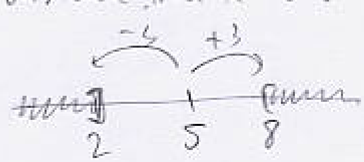
$19x=-67$

$x=-\frac{67}{19}$ is a solution, since it doesn't give a 0 in the denominator of the original equation

Solve the inequalities. Draw the solution and write it in interval form.

4. (6pts) $|x-5|\geq 3$

distance from x to $5 \geq 3$

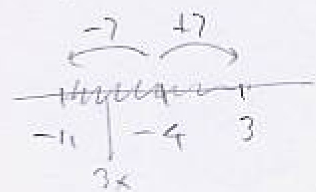


$(-\infty,2]\cup[8,\infty)$

5. (8pts) $|3x+4|<7$

$|3x-(-4)|<7$

distance from $3x$ to $-4<7$



$-11<3x<3$

$-\frac{11}{3}<x<1 \qquad \left(-\frac{11}{3},1\right)$

6. (12pts) The quadratic function $f(x) = x^2 - 4x + 7$ is given. Do the following without using the calculator.
a) Find the x- and y-intercepts of its graph, if any.
b) Find the vertex of the graph.
c) Sketch the graph of the function.

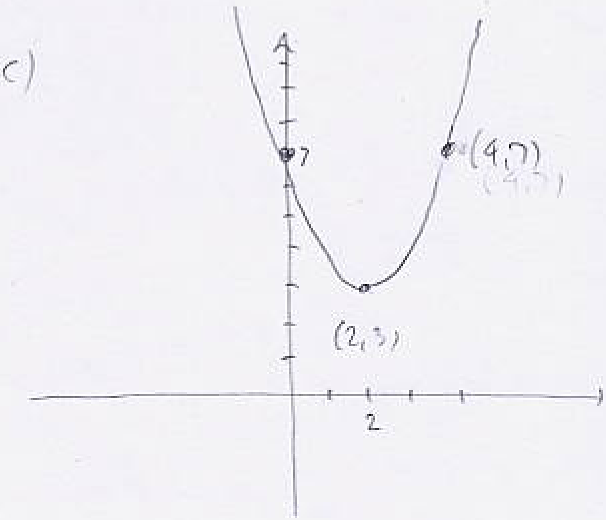
a) y-int : $f(0)=7$

x-int $x^2-4x+7=0$

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4\cdot7}}{2\cdot1}$$

$$=\frac{4\pm\sqrt{-12}}{2}$$ No real solution, so no intercepts.

c)



b) $h=-\frac{-4}{2\cdot1}=2$

$k=f(2)=2^2-4\cdot2+7=4-8+7=3$

Solve the equations:

7. (8pts) $4x^4 + 8x^2 - 5 = 0$

$4(x^2)^2+8x^2-5=0$

let $u=x^2$

$4u^2+8u-5=0$

$$u=\frac{-8\pm\sqrt{8^2-4\cdot4\cdot(-5)}}{2\cdot4}$$

$$=\frac{-8\pm\sqrt{64+80}}{8}=\frac{-8\pm\sqrt{144}}{8}$$

$$=\frac{-8\pm12}{2}=-\frac{20}{8},\frac{4}{8}=-\frac{5}{2},\frac{1}{2}$$

$x^2=-\frac{5}{2}$ $x=\pm\sqrt{\frac{5}{2}}i$

$x^2=\frac{1}{2}$ $x=\pm\sqrt{\frac{1}{2}}$

8. (8pts) $2 + \sqrt{2x-3} = \sqrt{x+7}$ $\Big|^2$

$2^2+2\cdot2\sqrt{2x-3}+\sqrt{2x-3}^2=\sqrt{x+7}^2$

$4+4\sqrt{2x-3}+2x-3=x+7$

$4\sqrt{2x-3}+2x+1=x+7$ $\Big|-2x-1$

$4\sqrt{2x-3}=6-x$ $\Big|^2$

$16(2x-3)=6^2-2\cdot6\cdot x+x^2$

$32x-48=36-12x+x^2$

$x^2-44x+84=0$

$(x-2)(x-42)=0$

$x=2$ or $x=42$

$2+\sqrt{4-3}\stackrel{?}{=}\sqrt{9}$ $\quad 2+\sqrt{81}\stackrel{?}{=}\sqrt{49}$

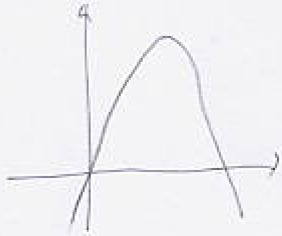
$2+1=3$ yes $\quad 2+3=7$ no

$\boxed{\text{sol } x=2}$

9. (12pts) A ball is thrown upwards with initial velocity 30 meters per second. The height of the ball in meters t seconds after release is given by the function $s(t) = -5t^2 + 30t$.
a) What is the maximum height that the ball achieves?
b) When does the ball hit the ground?

a) Graph of $s(t)$ is

$s(t) = -5t^2 + 30t$



Max. occurs when

$t = -\frac{30}{2\cdot(-5)} = 3$

$s(3) = -5\cdot 9 + 90 = 45\,m$

is max height

b) $-5t^2 + 30t = 0$

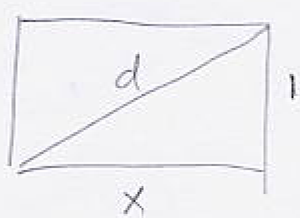
$-5t(t-6) = 0$

$t = 0, 6$

at release   at return

Hits ground at $t = 6s$

10. (14pts) The diagonal of a rectangle is $\frac{2}{5}$ the length of its perimeter. If one side of the rectangle has length 1, find the length of the other side .



$d^2 = x^2 + 1^2$

$d = \frac{2}{5}(2x + 2\cdot 1)$

$\left(\frac{2}{5}(2x+2)\right)^2 = x^2 + 1^2$

$\left(\frac{4}{5}(x+1)\right)^2 = x^2 + 1^2$

$\frac{16}{25}(x^2 + 2x + 1) = x^2 + 1$ 1.25

$16x^2 + 32x + 16 = 25x^2 + 25$

$9x^2 - 32x + 9 = 0$

$x = \frac{-(-32) \pm \sqrt{(-32)^2 - 4\cdot 9\cdot 9}}{2\cdot 9}$

$= \frac{32 \pm \sqrt{1024 - 364}}{18}$

$= \frac{32 \pm \sqrt{660}}{18}$

$= \frac{32 \pm 2\sqrt{115}}{18}$

$= \frac{16 \pm \sqrt{115}}{9}$

Both solutions are ≥ 0

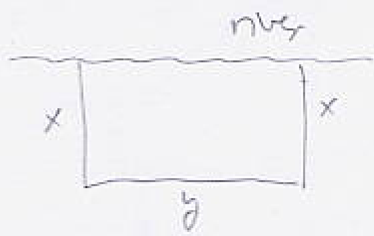
so will work.

11. (14pts) Farmer Dwayne has 250 meters of fencing that he will use to enclose a rectangular plot of land next to a straight river. The side of the rectangle along the river does not need fencing. Dwayne wishes to maximize the area of the rectangle.

a) Express the area of the enclosure as a function of the length of one of the sides. What is the domain of this function?

b) Sketch the graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What dimensions of the rectangle give you maximal area?

a)



$2x + y = 250$

$y = 250 - 2x$

$A = xy = x(250-2x)$

$A(x) = -2x^2 + 250x$

Domain: $x \geq 0$

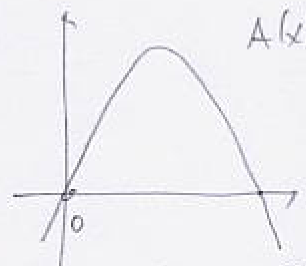
and $250 - 2x \geq 0$

$2x \leq 250$

$x \leq 125$

Domain $[0, 125]$

b)



$h = -\frac{250}{2\cdot(-2)} = \frac{250}{4} = 62.5$

Maximal area occurs for

$x = 62.5$

$y = 250 - 2\cdot 62.5 = 250 - 125 = 125$

62.5×125

Bonus. (10pts) Find the domain of the function $f(x) = \sqrt{|x-7| - |x+5|}$. *(Hint: use the "distance" interpretation of absolute value.)*

Must have $|x-7| - |x+5| \geq 0$

$|x-7| \geq |x-(-5)|$

dist. from x to 7 $\geq$ dist from x to -5

$\frac{-5+7}{2} = 1$