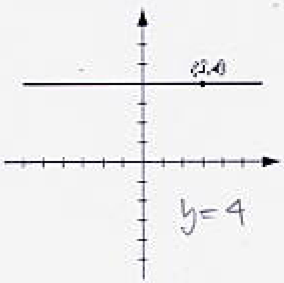
Algebra and Trigonometry — Exam 2
MAT 150, Fall 2013 — D. Ivanšić

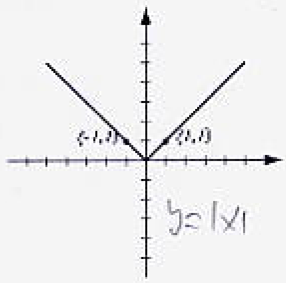
Name: Saul Ocean

Show all your work!

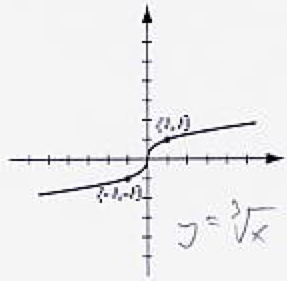
1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



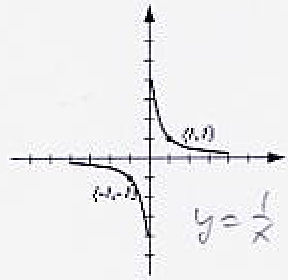
$y=4$



$y=|x|$



$y=\sqrt[3]{x}$



$y=\frac{1}{x}$

2. (6pts) Simplify, so that the answer is in form $a+bi$.

$$\frac{3+5i}{4-i} = \frac{3+5i}{4-i}\cdot\frac{4+i}{4+i} = \frac{12+3i+20i+5i^2}{4^2-i^2} = \frac{12+23i-5}{16-(-1)} = \frac{7+23i}{17} = \frac{7}{17}+\frac{23}{17}i$$

3. (4pts) Simplify and justify your answer.

$$i^{77} = i^{76}\cdot i = (\underbrace{i^4}_{=1})^{19}\cdot i = i$$

$76 = 4\cdot 19$ $\quad =1$

4. (8pts) Find the equation of the line (in form $y=mx+b$) that passes through the point $(3,-2)$ and is perpendicular to the line $3x-5y=3$. Draw both lines in the coordinate system.

$3x-5y=3$

$3x-3=5y$

$y=\frac{3x-3}{5}=\frac{3}{5}x-\frac{3}{5}$

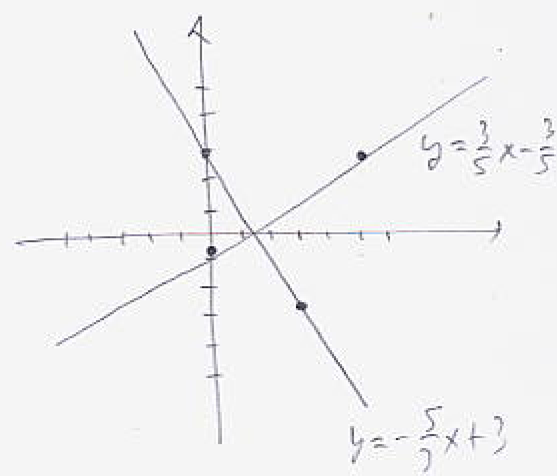
$m=\frac{3}{5}$

Perpendicular line

$y-(-2)=-\frac{5}{3}(x-3)$

$y+2=-\frac{5}{3}x+5$

$y=-\frac{5}{3}x+3$



5. (6pts) Solve the inequality. Write your solution in interval notation.

$-3 \leq 3x + 7 < 12$ $\quad (-7$

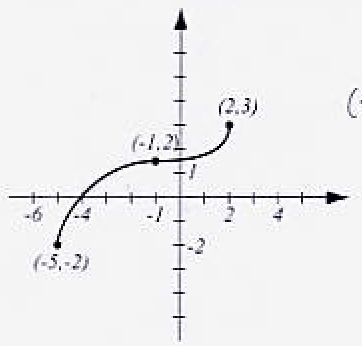
$-10 \leq 3x < 5 \quad (\div 3$

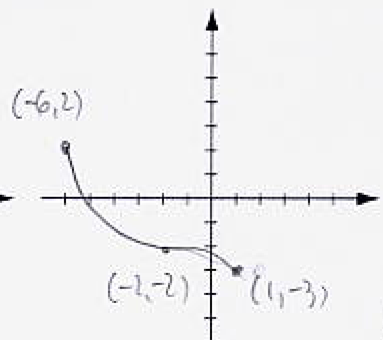
$-\frac{10}{3} \leq x < \frac{5}{3}$

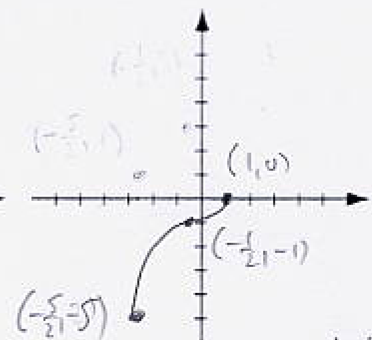
$-\frac{10}{3} \qquad \frac{5}{3}$

$\left[-\frac{10}{3}, \frac{5}{3}\right)$

6. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $-f(x+1)$ and $f(2x) - 3$ and label all the relevant points.







shift left 1
reflect in x-axis

Stretch horizontally, factor $= \frac{1}{2}$
shift down 3

7. (13pts) Let $f(x) = \dfrac{1}{x^2-4}$, $g(x) = 3x - 1$.

Find the following (simplify where possible):

$$\frac{f}{g}(x) = \frac{f(x)}{g(x)} = \frac{\frac{1}{x^2-4}}{3x-1} = \frac{1}{x^2-4} \cdot \frac{1}{3x-1}$$

$$= \frac{1}{(x^2-4)(3x-1)}$$

$$(f \circ g)(x) = f(g(x)) = f(3x-1)$$

$$= \frac{1}{(3x-1)^2-4} = \frac{1}{9x^2-6x+1-4}$$

$$= \frac{1}{9x^2-6x-3}$$

The domain of $\dfrac{f}{g}$

domain of f:
can't have $x^2-4=0$
$x^2=4$
$x=\pm 2$

domain of g:
all real numbers

Exclude where $g(x)=0$
$3x-1=0$
$3x=1$
$x=\frac{1}{3}$

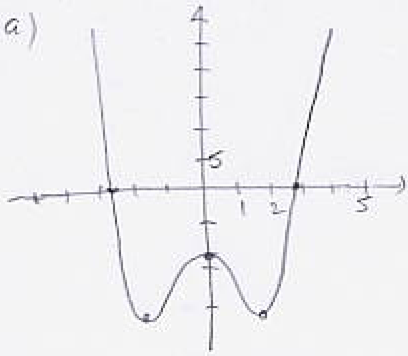
$-2 \qquad \frac{1}{3} \qquad 2$

$x \neq \pm 2, \frac{1}{3}$

Domain $= (-\infty,-2) \cup (-2,\frac{1}{3}) \cup (\frac{1}{3},2) \cup (2,\infty)$

8. (17pts) Let $f(x) = x^4 - 6x^2 - 8$ (answer with 6 decimal points accuracy).
a) Use your graphing calculator to accurately draw the graph of f (on paper!). Indicate scale on the graph.
b) Determine algebraically whether f is even, odd, or neither. Then verify your answer by looking at the graph (justify).
c) Find the local maxima and minima for this function.
d) State the intervals where the function is increasing and where it is decreasing.

a)



b) $f(-x) = (-x)^4 - 6(-x)^2 - 8$
$= x^4 - 6x^2 - 8 = f(x)$ so f is even
– agrees with graph, which is symmetric w.r.t. y-axis

c) local min: $f(1.732053) = -17$
$f(-1.732053) = -17$ (by symmetry)
local max: $f(0) = -8$

d) Increasing on $(-1.732053, 0) \cup (1.732053, \infty)$
Decreasing on $(-\infty, -1.732053) \cup (0, 1.732053)$

9. (14pts) Gina plans to invest \$12,000, part at 4% and the rest at 6% simple interest. What is the most that she can invest at 4% in order to get at least \$650 per year in interest?

x = amount invested at 4%

$12000 - x$ = amount invested at 6%

interest from 4% + interest from 6% ≥ 650

$0.04 \cdot x + 0.06(12000 - x) \geq 650$

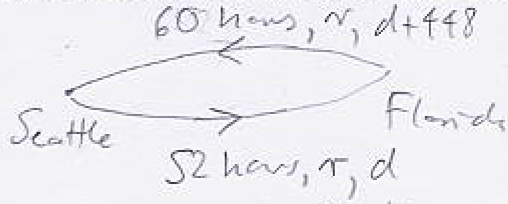
$0.04x + 720 - 0.06x \geq 650$

$-0.02x \geq -70$

$x \leq \frac{-70}{-0.02}$

$x \leq 3500$

10. (14pts) Fred, who is from Seattle, went to school in Florida. On the way to school, he took a southern route, and on his return after graduation, he took a northern route. On both trips he averaged the same speed. If the southern trek took 52 hours, the northern 60 hours, and the northern trek was 448 miles longer, how long was each trip?



r = Fred's speed

Southern: $d = 52 \cdot r$

Northern: $d + 448 = 60r$

d is same, so

$52r + 448 = 60r \quad | -52r$

$8r = 448$

$r = 56$ mph

$d = 52 \cdot 56 = 2912$

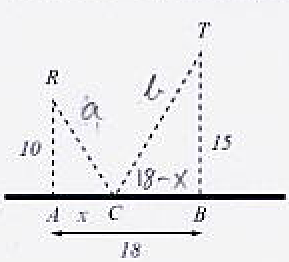
Southern route 2912 miles

Northern route $2912 + 448 = 3360$ miles

Bonus. (10pts) The towns of Rabbitton and Turtleville are 10 and 15 miles away from the nearest points A and B on an existing highway, which are 18 miles apart. Straight roads from the two towns are planned to join the highway at the same point C so that the total length of the connecting roads is minimized.

a) Express the total length of the connecting roads as a function of x, the distance between A and C. What is the domain of this function?

b) Graph the function in order to find the minimum. Where should the junction C be so that the total length of the connecting roads is the smallest?

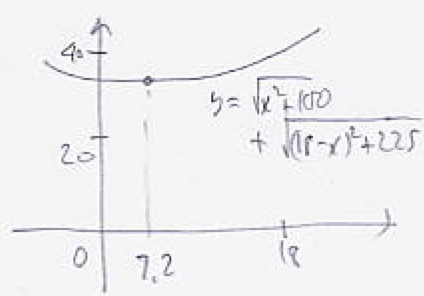


$L = a + b = \sqrt{x^2 + 10^2} + \sqrt{(18-x)^2 + 15^2} = L(x)$

$a^2 = 10^2 + x^2$

$b^2 = 15^2 + (18-x)^2$

Domain: $0 \le x \le 18$



Minimum length occurs for

$x = 7{,}2 \quad L(7{,}2) = 30{,}805844$