Algebra and Trigonometry — Exam 1
MAT 150, Fall 2013 — D. Ivanšić

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Show all your work!

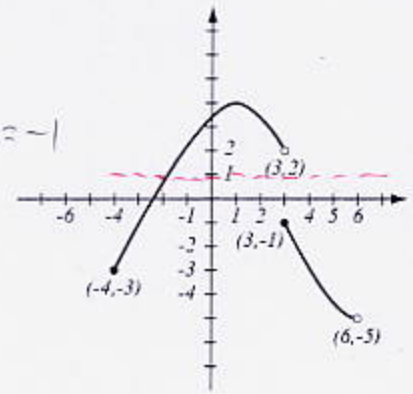
1. (8pts) Use the graph of the function f at right to answer the following questions.

a) Find $f(1)$ and $f(3)$. $f(1)=4,\ f(3)=-1$

b) What is the domain of f? $[-4,6)$

c) What is the range of f? $(-5,4]$

d) What are the solutions of the equation $f(x)=1$? $x=-1.8$



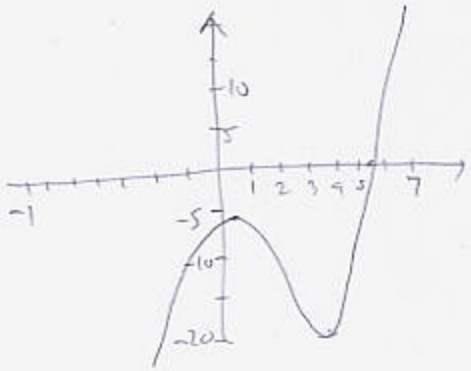
2. (10pts) The function $f(x)=x^3-6x^2+5x-7$ is given.

a) Use your calculator to accurately sketch its graph. Draw the graph here, and indicate units on the axes.

b) Find all the $x-$ and $y-$intercepts (accuracy: 6 decimal points).

y-int: $f(0)=-7$

x-int: $x=5.306334$



3. (4pts) Convert to scientific notation or a decimal number:

$0.000034612 = 3.4612\times10^{-5}$ $\qquad$ $7.347\times10^{4} = 73,470$

Use formulas to expand:

4. (4pts) $(2x-7y)(2x+7y) = (2x)^2-(7y)^2 = 4x^2-49y^2$

5. (4pts) $(x^3-2)^2 = (x^3)^2-2x^3\cdot2+2^2 = x^6-4x^3+4$

6. (6pts) $(x-4)^3 = x^3-3\cdot x^2\cdot4+3\cdot x\cdot4^2-4^3$

$= x^3-12x^2+48x-64$

Simplify, showing intermediate steps. Assume all variables are positive.

7. (2pts) $\sqrt{98} = \sqrt{49 \cdot 2}$

$= 7\sqrt{2}$

8. (5pts) $\sqrt[4]{162x^8y^5} = \sqrt[4]{2 \cdot 81 (x^2)^4 y^4 y}$

$= 3x^2 y \sqrt[4]{2y}$

9. (8pts) Express answers first in terms of positive exponents, then convert to root notation.

$$\frac{(16u^2v^{-3})^{\frac{3}{4}}}{3\left(u^{-3}v^{\frac{5}{4}}\right)^2} = \frac{16^{\frac{3}{4}}(u^2)^{\frac{3}{4}}(v^{-3})^{\frac{3}{4}}}{3\cdot(u^{-3})^2(v^{\frac{5}{4}})^2} = \frac{(\sqrt[4]{16})^3 u^{\frac{3}{2}} v^{-\frac{9}{4}}}{3u^{-6}v^{\frac{10}{4}}} = \frac{8}{3}u^{\frac{3}{2}-(-6)}v^{-\frac{9}{4}-\frac{10}{4}}$$

$$= \frac{8}{3}u^{\frac{15}{2}}v^{-\frac{19}{4}} = \frac{8u^{\frac{15}{2}}}{3v^{\frac{19}{4}}} = \frac{8\sqrt{u^{15}}}{3\sqrt[4]{v^{19}}}$$

10. (8pts) Simplify.

$$\frac{3x+1}{2x^2+x-6} - \frac{x+3}{x^2-3x-10} = \frac{3x+1}{(2x-3)(x+2)} - \frac{x+3}{(x-5)(x+2)}$$

prod $ac = -12$, sum 1: $4, -3$

$2x^2+4x-3x-6$

$2x(x+2)-3(x+2)$

$=(2x-3)(x+2)$

$$= \frac{(3x+1)(x-5)-(x+3)(2x-3)}{(2x-3)(x+2)(x-5)} = \frac{3x^2-14x-5-(2x^2+3x-9)}{(2x-3)(x+2)(x-5)}$$

$$= \frac{x^2-17x+4}{(2x-3)(x+2)(x-5)}$$ ← doesn't factor: prod $= 4$, sum $= -17$ } no such numbers

11. (6pts) Rationalize the denominator.

$$\frac{2\sqrt{3}-5}{\sqrt{3}-2} \cdot \frac{\sqrt{3}+2}{\sqrt{3}+2} = \frac{(2\sqrt{3}-5)(\sqrt{3}+2)}{(\sqrt{3})^2-2^2} = \frac{\overbrace{2\sqrt{3}^2}^{6}-5\sqrt{3}+4\sqrt{3}-10}{3-4} = \frac{-4-\sqrt{3}}{-1} = 4+\sqrt{3}$$

12. (8pts) Write the equation of a circle whose diameter has endpoints $A = (3, -2)$ and $B = (-5, 0)$.

Center = midpoint of $\overline{AB}$.

$= \left(\frac{3+(-5)}{2}, \frac{-2+0}{2}\right) = (-1, -1)$

radius $= \frac{d(A,B)}{2} = \frac{1}{2}\sqrt{(-5-3)^2 + (0-(-2))^2}$

$= \frac{1}{2}\sqrt{64+4} = \frac{1}{2}\sqrt{68}$

$= \frac{1}{2}\sqrt{4 \cdot 17} = \frac{2\sqrt{17}}{2} = \sqrt{17}$

$(x-(-1))^2 + (y-(-1))^2 = \sqrt{17}^2$

$(x+1)^2 + (y+1)^2 = 17$

13. (4pts) Solve the equation for b.

$A = \frac{1}{2}h(a+b)$ $\quad \cdot 2$

$2A = h(a+b) \quad \div h$

$\frac{2A}{h} = a+b \quad -a$

$b = \frac{2A}{h} - a$

14. (7pts) Let $g(x) = x^2 - 3x + 1$. Find the following (simplify where appropriate).

$g(-1) = (-1)^2 - 3(-1) + 1$

$= 5$

$g\left(\frac{a}{2}\right) = \left(\frac{a}{2}\right)^2 - 3\frac{a}{2} + 1 = \frac{a^2}{4} - \frac{3a}{2} + 1$

$g(x-4) = (x-4)^2 - 3(x-4) + 1 = x^2 - 8x + 16 - 3x + 12 + 1$

$= x^2 - 11x + 29$

15. (6pts) Find the domain of the function $f(x) = \dfrac{\sqrt{x}}{x^2 - x - 12}$ and write it in interval notation.

Can't have

$x^2 - x - 12 = 0$

$(x-4)(x+3) = 0$

$x = -3, 4$

Must have

$x \geq 0$

Domain $= \{x \mid x \geq 0 \text{ and } x \neq 4\}$

$= [0, 4) \cup (4, \infty)$

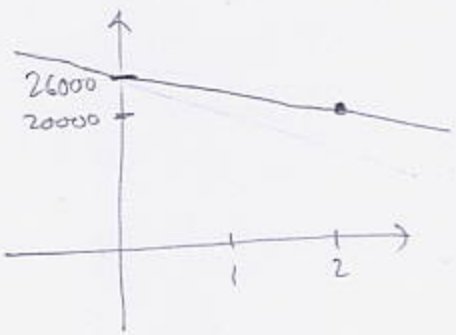
16. (10pts) A business purchases a car for \$26,000. For tax purposes, it is assumed that the car will lose \$3000 in value every year.
a) Write the linear function $V(t)$ that expresses the value of the car after t years.
b) Graph the function $V(t)$ and find $V(5)$.
c) What is the meaning of the slope of the line $V(t)$?

a) $V(t) = 26000 - 3000t$

c) The slope is the annual loss of value of the car:

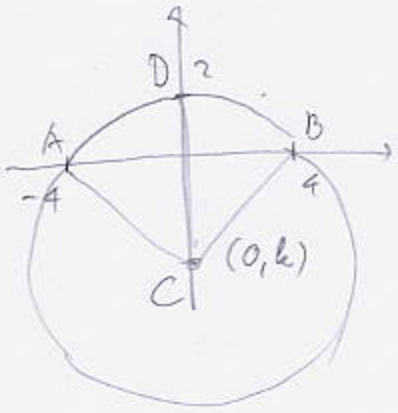
-3000 \$/yr

b)



$V(5) = 26000 - 3000 \cdot 5$
$= 11000$

Bonus (10pts) Find the equation of the circle that passes through points $(-4, 0)$, $(4, 0)$ and $(0, 2)$. *Hint: due to symmetry, the center must be on the y-axis.*



$d(A,C) = d(D,C)$

$\sqrt{(0-(-4))^2+(k-0)^2} = \sqrt{(0-0)^2+(k-2)^2}$ |²

$k^2 + 16 = k^2 - 4k + 4$ |−4

$-4k = 12$

$k = -3$

Center $(0,-3)$

Radius $= d(C,D) = 5$

$(x-0)^2 + (y-(-3))^2 = 5^2$

$x^2 + (y+1)^2 = 25$