1. (3pts) For the matrices A, B and C find the following expressions, if they are defined:
a) ABC b) BB^TC

$$A = \begin{bmatrix} 2 & 1 & -1 \end{bmatrix}$$

$$B = \begin{bmatrix} -1 & 1 \\ 3 & 2 \\ 2 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$$

2. (8pts) A system of linear equations is given below.
a) Use the Gauss-Jordan method (that is, transform the augmented matrix to reduced row-echelon form) in order to solve the system.
b) Write the solution in vector form.
c) Write the solution of the homogeneous system (numbers on the right replaced by 0's). What is the basis of this subspace? What is the dimension?

$$\begin{array}{rrrrl} x_1 & -x_2 & +2x_3 & -x_4 & = -1 \\ 2x_1 & +x_2 & -2x_3 & -2x_4 & = -2 \\ -x_1 & +2x_2 & -4x_3 & +x_4 & = 1 \\ 3x_1 & & & -3x_4 & = -3 \end{array}$$

3. (4pts) Evaluate the determinant by any (efficient) method:

$$\begin{vmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{vmatrix} =$$

4. (6pts) The matrix A is given below.
a) Find A^{-1}.
b) Use the result of a) to easily solve the system $A\mathbf{x} = \mathbf{b}$, where $\mathbf{b} = (-1, 2, 4)$.

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

5. (8pts) The matrix A is given below.
a) Find the eigenvalues for the matrix.
b) For each eigenvalue, find a corresponding eigenvector.
c) Is there a basis of $\mathbf{R}^2$ consisting entirely of eigenvectors of A?

$$A = \begin{bmatrix} -5 & 7 \\ 1 & 3 \end{bmatrix}$$

6. (4pts) Are the vectors $(4, 1, 3)$, $(-2, 1, 8)$ and $(0, 1, -5)$ a basis for $\mathbf{R}^3$?

7. (4pts) Do the vectors of form (a, b, c) where $a + b + c = 1$ form a subspace of $\mathbf{R}^3$? Justify your answer.

8. (4pts) Find the matrix of the linear operator $T : \mathbf{R}^3 \to \mathbf{R}^3$ that is the composition of a rotation by $60°$ about the positive x-axis, followed by a projection to the xz-plane.

9. (5pts) Let T be the rotation about the origin in $\mathbf{R}^2$ by $30°$. Find the vector that T sends to the vector $(-5, 3)$.

10. (8pts) Let A be a 5×3 matrix. Answer the following and justify your answers.
a) What is the biggest rank(A) could be?
b) What is the smallest nullity(A) could be?
c) If T_A is the linear transformation corresponding to A, is T_A ever onto? Is it ever one-to-one?

11. (3pts) Let E_1 be the matrix obtained from I_2 by adding 3 times row 1 to row 2 and let E_2 be be the matrix obtained from I_2 by swapping the two rows. Find the matrix below.

$$E_2E_1\begin{bmatrix} 3 & -8 \\ 11 & -2 \end{bmatrix} =$$

12. (4pts) Let W be the subspace of $\mathbf{R}^4$ spanned by vectors $(1,2,1,4)$ and $(3,1,-1,0)$. Find a basis for $W^{\perp}$.

13. (9pts) Are the following statements true or false? Justify your answer by giving a logical argument or a counterexample.
a) For $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbf{R}^3$, if $\mathbf{u} \cdot \mathbf{v} = 0$ and $\mathbf{v} \cdot \mathbf{w} = 0$, then $\mathbf{u} \cdot \mathbf{w} = 0$.
b) If $T : \mathbf{R}^2 \to \mathbf{R}^2$ is a linear operator, then $T(\mathbf{x} \cdot \mathbf{y}) = \mathbf{x} \cdot \mathbf{y}$ for every $\mathbf{x}$, $\mathbf{y}$ in $\mathbf{R}^2$.
c) If A is an $n \times n$ matrix, then $\mathbf{0}$ is the only vector that is both in row(A) and null(A).

Bonus. (7pts) Let S be the set of vectors $S = \{(1, 2, -1), (4, 9, -6), (3, 7, -5), (6, 13, -8)\}$.
a) Find a basis for span(S) that consists only of vectors in S.
b) Complete the basis you found in a) to a basis of $\mathbf{R}^3$.