1. (5pts) Evaluate the determinant by any (efficient) method:

$$\begin{vmatrix} 3 & 2 & 3 & -1 & -4 \\ 4 & 7 & 2 & 3 & 15 \\ 0 & 0 & 2 & 5 & 2 \\ 0 & 0 & -4 & -3 & 0 \\ 0 & 0 & 2 & -1 & 4 \end{vmatrix} =$$

2. (3pts) If $\det A = -5$ and A is a 2×2 matrix, find:

$\det A^{-1} =$

$\det(3A) =$

$\det A^4 =$

3. (6pts) Let $A\mathbf{x} = \mathbf{b}$ be a linear system whose solution is given below (A is a 2×4 matrix).
a) Write any two solutions of the system.
b) Write the general solution of the system $A\mathbf{x} = \mathbf{0}$.
c) State the vectors that span the solution space of $A\mathbf{x} = \mathbf{0}$.

$$\begin{aligned} x_1 &= 3 - 2s + 4t \\ x_2 &= 7 + 3s \\ x_3 &= -1 + 8s - 7t \\ x_4 &= -5s + t \end{aligned}$$

4. (6pts) Determine whether the vectors $(1, 3, 2)$, $(-2, 0, 7)$ and $(5, 3, -12)$ are linearly independent. Then draw a picture of these vectors that captures their relative positions to one another. Do not pay attention to actual coordinates.

5. (6pts) The matrix A is given below.
a) Find the eigenvalues for the matrix.
b) For each eigenvalue, find a corresponding eigenvector.

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 5 \end{bmatrix}$$

6. (4pts) Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be the rotation about the origin by $120°$.
a) Write the standard matrix of this transformation.
b) Find $T(1, 3)$.

7. (7pts) Write the standard matrices for the following linear operators.
a) $T : \mathbf{R}^2 \to \mathbf{R}^2$, T dilates by 4 in the x-direction, then reflects in the line $y = x$.
b) $T : \mathbf{R}^3 \to \mathbf{R}^3$, T rotates about the positive z-axis by $90°$, then reflects in the xz-plane.

8. (4pts) Show that the set of vectors of form $(a, b, 3a - 2b)$ is a subspace of $\mathbf{R}^3$.

9. (9pts) Are the following statements true or false? Justify your answer by giving a logical argument or a counterexample.

a) If $\det A = 0$, then $\lambda = 1$ cannot be an eigenvalue of A.

b) If A is orthogonal, then $\det A \neq 0$.

c) If $T : \mathbf{R}^2 \to \mathbf{R}^2$ is a linear operator, then $T(\mathbf{x} \cdot \mathbf{y}) = \mathbf{x} \cdot \mathbf{y}$ for every $\mathbf{x}$, $\mathbf{y}$ in $\mathbf{R}^2$.

Bonus. (5pts) Show that

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = (y-x)(z-x)(z-y)$$