1. (6pts) For the matrices A, B and C find the following expressions, if they are defined:
a) A^2C b) BB^T c) $2C - BA$

$$A = \begin{bmatrix} 2 & 1 \\ -1 & -1 \\ 0 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 7 & 0 & 1 \\ -2 & 3 & 2 \end{bmatrix}$$

$$C = \begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}$$

2. (6pts) The matrix A is given below.
a) Find the inverse of A.
b) Use the inverse to effortlessly solve the system below.

$$A = \begin{bmatrix} 2 & 4 \\ 7 & -1 \end{bmatrix}$$

$$\begin{array}{lll} 2x_1 & +4x_2 & = 1 \\ 7x_1 & -x_2 & = 3 \end{array}$$

3. (4pts) Find the cosine of the angle between the vectors $\mathbf{a} = (1, -1, 3, 4)$ and $\mathbf{b} = (0, 4, 5, 2)$.

4. (9pts) A system of linear equations is given below.
a) Use the Gauss-Jordan method (that is, transform the augmented matrix to reduced row-echelon form) in order to solve the system.
b) Write the solution in vector form.
c) Describe the set of points in $\mathbf{R}^4$ that the solution set represents.

$$\begin{array}{rrrrl} 3x_1 & +x_2 & & +13x_4 & = 11 \\ & -x_2 & -x_3 & -6x_4 & = -1 \\ 2x_1 & +2x_2 & +x_3 & +17x_4 & = 9 \end{array}$$

5. (5pts) Below is the augmented matrix of a system of linear equations. Determine the c's for which the system has: a) one solution, b) infinitely many solutions, c) no solutions. (Note: no row operations are needed.)

$$A = \begin{bmatrix} 1 & 3 & 4 & 5+c \\ 0 & 1 & -17 & 7 \\ 0 & 0 & c^2-4c & c-4 \end{bmatrix}$$

6. (3pts) The matrix B was obtained by applying a row operation to matrix A. Find the elementary matrix E so that $EA = B$.

$$A = \begin{bmatrix} 3 & 7 & -7 \\ 1 & -6 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 7 & -17 & 9 \\ 1 & -6 & 4 \end{bmatrix} \qquad E =$$

7. (3pts) Find a 2×2 matrix B so that $B \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 3a & 3b \\ 5a & 5b \end{bmatrix}$ for every 2×2 matrix.

8. (4pts) Suppose we have a system with 4 equations in 3 unknowns. Every equation represents a plane $\mathbf{R}^3$. Draw one example of a 4-plane arrangement for each of the following situations: a) the system has no solution b) the solution is a line in $\mathbf{R}^3$.

9. (10pts) Are the following statements true or false? Justify your answer by giving a logical argument or a counterexample.

a) If $\mathbf{u}$ is orthogonal to $\mathbf{v}$, then $\|\mathbf{u}+\mathbf{v}\|^2 = \|\mathbf{u}-\mathbf{v}\|^2$

b) If A is a 3×5 matrix with at least 2 non-zero entries, then the solution set of the linear system $A\mathbf{x} = \mathbf{0}$ always has at most 3 parameters.

c) If A is an $n \times n$ matrix and $A^{17} = I$, then A is invertible.

Bonus. (5pts) Use a linear system to show that the vector $(-2, 25, 11)$ is a linear combination of vectors $(2, 1, 3)$ and $(3, -5, 1)$ and find the coefficients that realize this linear combination.