

<p>Calculus 1 — Exam 7 MAT 250, Spring 2011 — D. Ivanšić</p>
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Name: _____
Show all your work!

Find the following antiderivatives.

1. (4pts) $\int \cos\left(5x - \frac{\pi}{2}\right) dx =$

2. (7pts) $\int (x^3 - 4x)\sqrt{x} dx =$

3. (5pts) $\int \frac{1}{1 + (3x)^2} dx =$

4. (16pts) Find $\int_{-1}^3 |x| dx$ in two ways (they'd better give you the same answer!):

a) Using the “area” interpretation of the integral. Draw a picture.

b) Using the Fundamental Theorem of Calculus (you will have to break it up into two integrals).

5. (6pts) Write in sigma notation.

$$\frac{1}{2} - \frac{2}{3} + \frac{3}{4} - \frac{4}{5} + \frac{5}{6} =$$

Use the substitution rule in the following integrals:

6. (9pts) $\int \frac{4x^3 + 14x}{\sqrt[5]{x^4 + 7x^2 + 9}} dx =$

7. (10pts) $\int_{\ln 3}^{\ln 5} \frac{e^x}{(7 + e^x)^2} dx =$

8. (6pts) $\int_0^4 \frac{x - 2}{\sin(x^2 - 4x + 7)} dx =$

9. (8pts) A rocket shoots up vertically with velocity $v(t) = 5t^3 + 4t^2$ (in meters/second). Find its position function $s(t)$, if at time $t = 6$, the rocket is at altitude 1800m.

10. (21pts) The function $f(x) = e^x$, $0 \leq x \leq 3$ is given.

a) Write down the expression that is used to compute M_6 . Then compute M_6 .

b) Illustrate with a diagram, where appropriate rectangles are clearly visible. What does M_6 represent?

c) Using the Fundamental Theorem of Calculus, evaluate $\int_0^3 e^x dx$. Is M_6 an overestimate or an underestimate of this integral?

11. (8pts) Show that $\frac{\pi}{3} \leq \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \sin x \, dx \leq \frac{2\pi}{3}$ **without** evaluating the integral.

Bonus. (10pts) The rate at which money flows in or out of a company's account is given by the formula $3t^2 - 48$ dollars/day, $0 \leq t \leq 30$, t in days. At time $t = 0$, there was \$800 in the account.

- When is the company losing money from the account, and when is it gaining money?
- By how much did the amount in the account increase or decrease from $t = 0$ to $t = 6$?
- How much money is in the account when $t = 6$?