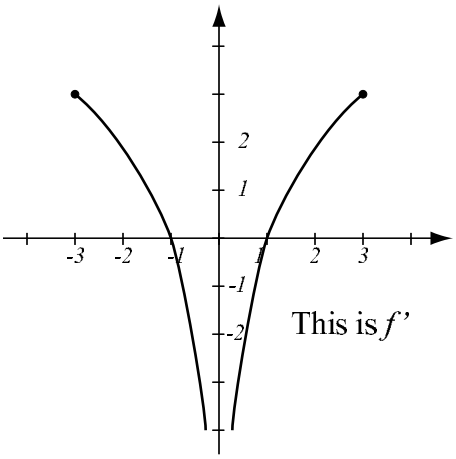
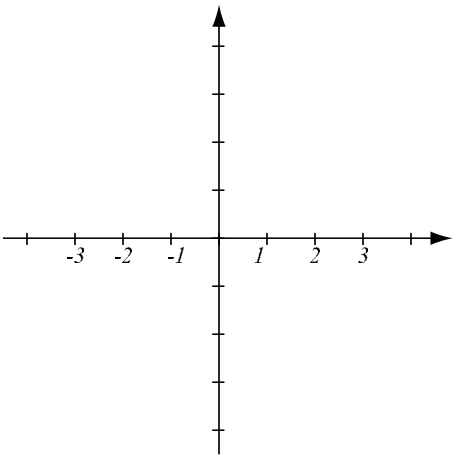
Calculus 1 — Exam 5
MAT 250, Spring 2011 — D. Ivanšić

Name: _____

Show all your work!

1. (18pts) Let f be continuous on $[-3, 3]$. The graph of its derivative f' is drawn below. Note that f' is not defined at 0. Use the graph to answer:
a) What are the intervals of increase and decrease of f? Where does f have a local minimum or maximum?
b) What are the intervals of concavity of f? Where does f have inflection points?
c) Use the information gathered in a) and b) to draw one possible graph of f at right. Note that f *is* defined at 0, make it, for example, $f(0) = 1$.





2. (12pts) Use Rolle's Theorem to show that the equation $2x + \sin x = 0$ has at most one solution.

3. (12pts) Verify the Mean Value Theorem for the function $f(x) = \ln x$ on the interval $[1, e^2]$. (Approximate $e \approx 3$ when necessary).

4. (22pts) Let $f(x) = \sin^4 x + \cos^4 x$, where $x \in [0, \pi]$.
a) Find the critical points and the intervals of increase and decrease of f. Determine where f has a local maximum or minimum.
b) Using information found in a), draw a rough sketch of f.

5. (14pts) Let $f(x) = \dfrac{x+3}{x^2+7}$. Find the absolute minimum and maximum values of f on the interval $[-1, 2]$.

6. (22pts) Let $f(x) = x^2 e^x$.
a) Find the intervals of concavity and points of inflection for f.
b) Find the critical points and use a) to determine which are local minima or maxima.

Bonus. (10pts) Let f be differentiable for all real numbers x so that $m_1 \leq f'(x) \leq m_2$ and $f(a) = b$. Use the Mean Value Theorem to show that the graph of f must be between the two lines with slopes m_1 and m_2, shown in the picture.