Calculus 1 — Exam 4 MAT 250, Spring 2011 — D. Ivanšić

Name:

Show all your work!

Differentiate and simplify where appropriate:

1. (4pts)
$$\frac{d}{dx}\sqrt{x^3-4x^2+1} =$$

2. (6pts)
$$\frac{d}{dx}e^{3x}\sin 5x =$$

3. (7pts)
$$\frac{d}{du} e^{\frac{1}{u^2+5u}} =$$

4. (8pts)
$$\frac{d}{dx} \ln \frac{x^2 + 1}{(x+4)^2} =$$

5. (6pts)
$$\frac{d}{dy} 7^{7^y} =$$

6. (9pts)
$$\frac{d}{dx} \frac{\arccos x}{\sqrt{1-x^2}} =$$

7. (12pts) Use implicit differentiation to find the equation of the tangent line to the ellipse $\frac{x^2}{25} + \frac{y^2}{4} = 1$ at the point $(\sqrt{5}, \frac{4}{\sqrt{5}})$. Draw the picture of the ellipse and the tangent line.

8. (10pts) Use implicit differentiation to find y'.

$$x^4 + y^4 = \frac{x}{y}$$

9. (10pts) Use logarithmic differentiation to find the derivative of $y = (\arctan x)^x$.

10. (12pts) Let $f(x) = x^2 - 4x$, $x \ge 2$, and let g be the inverse of f. Use the theorem on derivatives of inverses to find g'(12).

11. (16pts) A workers' platform raises as point B is pulled by a chain towards the stationary point A (circles denote rotating joints). If the diagonal beams are 5 meters long, and B is pulled towards A at rate 2 meters per minute, at what rate is the top of the platform rising when B is 3 meters away from A? What are the units? (*Hint: use a triangle.*)



Bonus. (10pts) In the previous problem let θ be the angle between either of the diagonal beams and the horizontal. Find how fast θ is increasing at the moment described above.