

Calculus 1 — Exam 2
MAT 250, Spring 2011 — D. Ivanšić

Name: _____
Show all your work!

1. (17pts) Differentiate and simplify where appropriate:

$$\frac{d}{dx} \left(4x^3 - \frac{3}{x^6} + \sqrt[3]{x^{13}} + 4^5 \right) =$$

$$\frac{d}{dx} ((\sqrt{x} + 7)(3\sqrt[4]{x} - x^2)) =$$

$$\frac{d}{dy} (4e^y + 3e^4) =$$

2. (10pts) Use the Intermediate Value Theorem to show that the equation $\cos x = x - 1$ has at least one solution. Write a nice sentence that shows how you are using the IVT.

3. (22pts) Find the following limits algebraically.

$$\lim_{x \rightarrow -4} \frac{x^2 - 3x - 28}{x^2 - 16} =$$

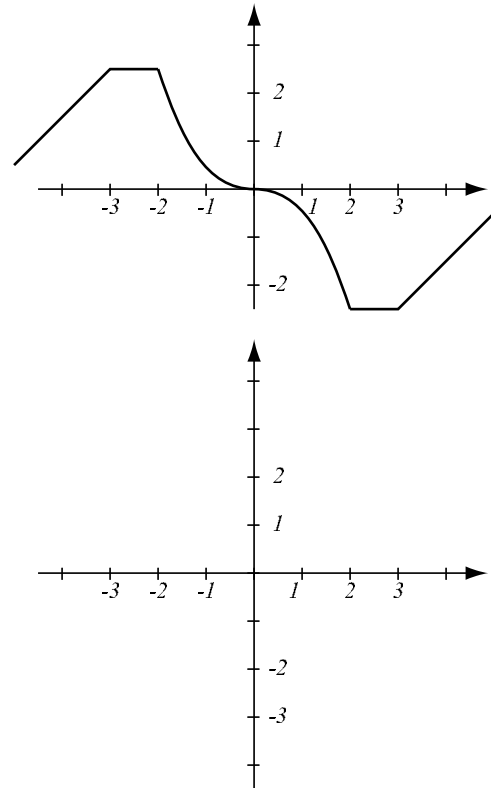
$$\lim_{x \rightarrow 1} \frac{\sqrt{x+3} - 2}{x-1} =$$

$$\lim_{x \rightarrow 0} \frac{\sin(7x)}{\sin(5x)} =$$

4. (10pts) Find $\lim_{x \rightarrow 0} x^2 \left(\sin \frac{1}{x} + \cos \frac{1}{x} \right)$. Use the theorem that rhymes with what you pay at the bursar's office, other than tuition.

5. (15pts) The graph of the function $f(x)$ is shown at right.

- Find the points where $f'(a)$ does not exist.
- Use the graph of $f(x)$ to draw an accurate graph of $f'(x)$.
- Is $f(x)$ odd or even? How about $f'(x)$?



6. (16pts) Let $f(x) = \frac{1}{x^2}$.

- Use the limit definition of the derivative to find the derivative of the function.
- Check your answer by taking the derivative of f using rules.
- Write the equation of the tangent line to the curve $y = f(x)$ at point $(2, \frac{1}{4})$.

7. (10pts) Consider the limit below. It represents a derivative $f'(a)$.

a) Find f and a .

b) Use the information above to find the limit.

$$\lim_{h \rightarrow 0} \frac{\sqrt[3]{8+h} - 2}{h}$$

Bonus. (10pts) Use the limit definition of the derivative to find the derivative of the function

$$f(x) = \frac{x^2}{x+3}.$$