Calculus 1 — Exam 1
MAT 250, Spring 2012 — D. Ivanšić

Name: Saul Ocean

Show all your work!

1. (16pts) Use the graph of the function to answer the following. Justify your answer if a limit does not exist.

$\lim_{x \to -1^-} f(x) = -\infty$

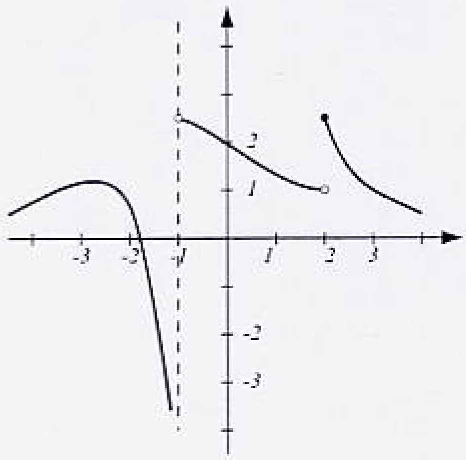
$\lim_{x \to -1^+} f(x) = 2.5$

$\lim_{x \to 2-} f(x) = 1$

$\lim_{x \to 2^+} f(x) = 2.5$ (one-sided limits are not equal)

$\lim_{x \to 2} f(x) =$ d.n.e

$f(2) = 2.5$



List points where f is not continuous and justify why it is not continuous at those points.

f is not continuous at $x=-1$ (not defined there)

$x = 2$ ($\lim_{x \to 2} f(x)$ d.n.e.)

2. (4pts) Find the following limit algebraically (no need to justify, other than showing the computation).

$\lim_{x \to 2} \cos(x^2 + 5x - 14) = \cos(2^2 + 5 \cdot 2 - 14) = \cos 0 = 1$

3. (8pts) Let $\lim_{x \to 5} f(x) = -7$ and $\lim_{x \to 5} g(x) = 3$. Use limit laws to find the limit below and show each step.

$$\lim_{x \to 5} \frac{x - 3g(x) - 2}{f(x) \cdot g(x) + x} = \frac{\lim_{x\to5}(x - 3g(x) - 2)}{\lim_{x\to5}(f(x)\cdot g(x) + x)} = \frac{\lim_{x\to5} x - \lim_{x\to5}(3g(x)) - \lim_{x\to5} 2}{\lim_{x\to5}(f(x)\cdot g(x)) + \lim_{x\to5} x}$$

$$= \frac{\lim_{x\to5} x - 3\lim_{x\to5} g(x) - 2}{\lim_{x\to5} f(x) \cdot \lim_{x\to5} g(x) + \lim_{x\to5} x} = \frac{5 - 3 \cdot 3 - 2}{-7 \cdot 3 + 5} = \frac{-6}{-16} = \frac{3}{8}$$

4. (14pts) Let $f(x) = \dfrac{3x^2 + x - 1}{x - 3}$.

a) Find the domain of f.

b) Explain, using continuity laws, why the function is continuous on its domain.

c) At points of discontinuity, state the type of discontinuity (jump, infinite, removable) and justify.

a) $x - 3 = 0$

$x = 3$

Domain $= \{x \mid x \neq 3\}$

$= (-\infty, 3) \cup (3, \infty)$

c) $\lim\limits_{x \to 3^+} \dfrac{3x^2 + x - 1}{x - 3} = \dfrac{27 + 3 - 1}{0+} = \dfrac{29}{0+} = \infty$

At $x = 3$ $f(x)$ has an infinite discontinuity.

b) $3x^2 + x - 1$ is continuous on $\mathbb{R}$ as a polynomial

$x - 3$ is " $\mathbb{R}$

$f(x)$ is cont. as a quotient of continuous functions

5. (16pts) The temperature in degrees Celsius of a rotisserie chicken left to cool is given by $f(t) = 0.15t^2 - 6t + 100$, where t is in minutes.

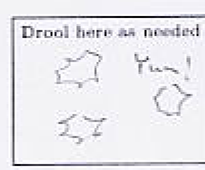
a) Find the average rates of change of temperature of the chicken over six short intervals of time, three of them beginning with 4, and three ending with 4. Show the table of values. What are the units?

b) Use the information in a) to find the instantaneous rate of cooling of the chicken at $t = 4$. What are the units?

a) average rate of change over $[4, t]$ $= \dfrac{f(t) - f(4)}{t - 4}$

$f(4) = 0.15 \cdot 16 - 24 + 100$
$= 78.4$

$= \dfrac{0.15t^2 - 6t + 100 - 78.4}{t - 4} = \dfrac{0.15t^2 - 6t + 21.6}{t - 4}$



Interval	Average ROC
[4, 4.1]	-4.785
[4, 4.01]	-4.7985
[4, 4.001]	-4.79985
[3.9, 4]	-4.815
[3.99, 4]	-4.8015
[3.999, 4]	-4.80015

units: °C/min

b) It appears that inst. rate of cooling is 4.8 °C/min

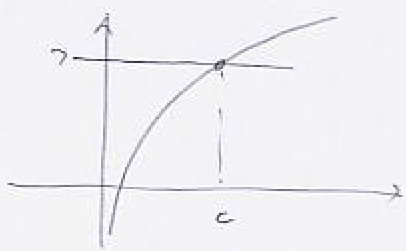
6. (14pts) Find an interval of length at most 0.01 that contains the solution of the equation $\ln x = 7 - x$. Use the Intermediate Value Theorem to justify why your interval contains the solution.

$\ln x = 7 - x$

$\ln x + x = 7$

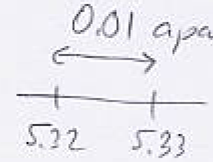
Let $f(x) = \ln x + x$.

It is a continuous function



By tracing, we find candidate endpoints of an interval:

x	$f(x)$
5.32	6.9914
5.33	7.0967



Since 7 is between $f(5.32)$ and $f(5.33)$, there is a $c \in (5.32, 5.33)$ such that $f(c) = 7$.

7. (16pts) Consider the limit $\lim_{x\to 0} \dfrac{\cos x - 1}{x^2}$.

a) Can you use a limit law to find the limit? Why or why not?

b) Fill out the table of values below. What do you think is the limit to six decimals accuracy?

c) Explain the unusual values at the bottom of the table.

x	$\dfrac{\cos x - 1}{x^2}$
0.1	-0.499583
0.01	-0.4999958
0.001	-0.49999996
10^{-4}	-0.5
10^{-5}	-0.5
10^{-6}	0
10^{-7}	0

a) $\lim_{x\to 0} \dfrac{\cos x - 1}{x^2} = \dfrac{\lim_{x\to 0}(\cos x - 1)}{\lim_{x\to 0} x^2} = \dfrac{0}{0}$

May not use limit laws because the limit in the denominator is 0.

b) It appears to be -0.5

c) For x close to 0, $\cos x$ is close to 1. If $\cos x > 1 - 10^{-13}$, the calculator will store it as 1, since it holds only 13 significant digits of a number. Then $\cos x - 1 = 0$ to the calculator, producing $\dfrac{\cos x - 1}{x^2} = 0$.

8. (12pts) Draw the graph of a function, defined on the interval $(-3, 4)$ that exhibits the following features:

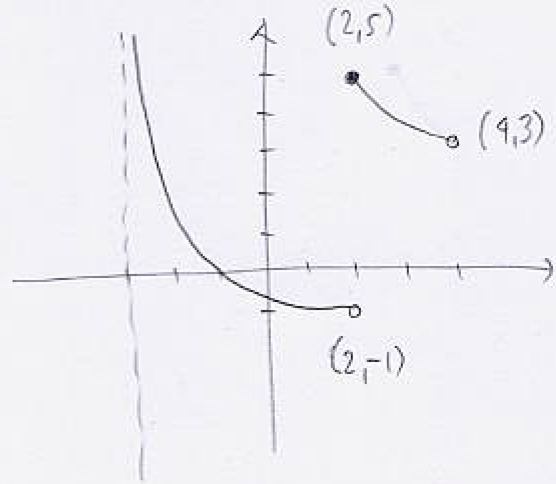
$\lim_{x \to -3^+} f(x) = \infty$

$\lim_{x \to 4^-} f(x) = 3$

$\lim_{x \to 2^-} f(x) = -1$

$\lim_{x \to 2^+} f(x) = 5$

$f(x)$ is right-continuous at $x = 2$



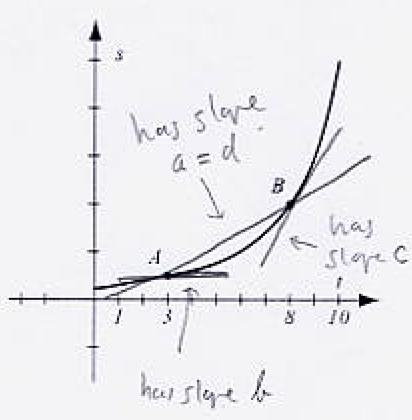
Bonus. (10pts) Below is the graph of the position of a car t minutes after noon. Arrange the numbers a, b, c and d in increasing order (some may be equal) and justify.

a =slope of secant line through points A and B

b =instantaneous velocity at time $t = 3$

c =slope of tangent line at point B.

d =average velocity over the interval $[3, 8]$.



Recall that on the graph of position:

average velocity over $[t_1, t_2]$ = slope of secant line through $(t_1, s(t_1))$, $(t_2, s(t_2))$

inst. velocity at t = slope of tangent line at $(t, s(t))$

By comparing slopes of lines, we see

$$b < \underbrace{a, d}_{a=d} < c$$