**1.** (12pts) When rolling a single die multiple times, what is the probability of
a) rolling a 2 twice on two tries?
b) not rolling a 5 on any of four tries?
c) rolling a 3 at least once on six tries?

a) $P(2 \text{ on 1st and 2 on 2nd})$
$= P(2 \text{ on 1st}) \cdot P(2 \text{ on 2nd}) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36} = 0.027778$

b) $P(\text{not 5 on 1st AND not 5 on 2nd AND ...})$
$= P(\text{not 5 on 1st}) \cdot P(\text{not 5 on 2nd}) \cdot ... P(\text{not 5 on 4th})$
$= \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} = \frac{625}{1296} = 0.482253$

c) $P(\text{rolling 3 at least once}) = 1 - P(\text{not rolling 3 on any of six tries})$
$= 1 - \left(\frac{5}{6}\right)^6 = 1 - \frac{15625}{46656} = \frac{31031}{46656} = 0.665102$

**2.** (14pts) A bag contains 15 yellow M&M's and 9 red M&M's. Two pieces are drawn without replacement. What is the probability that:
a) the second is red, given that the first one is yellow?
b) both are red?
c) first is red, second is yellow?
d) at least one is red?

24 total

a) $P(\text{2nd red} \mid \text{1st yellow}) = \frac{9}{23} = 0.391309$

b) $P(\text{1st red and 2nd red}) = P(\text{1st red}) \cdot P(\text{2nd red} \mid \text{1st red})$
$= \frac{3 \cancel{9}}{\cancel{1 3} \cancel{24}} \cdot \frac{\cancel{8}^{1}}{23} = \frac{3}{23} = 0.130435$

c) $P(\text{1st red and 2nd yellow}) = P(\text{1st red}) \cdot P(\text{2nd yellow} \mid \text{1st red})$
$= \frac{3 \cancel{9}}{8 \cancel{24}} \cdot \frac{15}{23} = \frac{45}{184} = 0.244565$

d) $P(\text{at least one is red}) = 1 - P(\text{both are yellow})$
$= 1 - P(\text{1st yellow}) \cdot P(\text{2nd yellow} \mid \text{1st yellow})$
$= 1 - \frac{5 \cancel{15}}{\cancel{8} \cancel{24}} \cdot \frac{14^{7}}{23} = 1 - \frac{35}{92} = \frac{57}{92} = 0.619565$

**3.** (10pts) The table shows the pattern of beverage purchases at a certain grocery store one fine Friday. What is the probability that a random shopper:
a) bought soda?
b) was 50 and over?
c) was 31–49 and bought soda?
d) bought soda, given they were 31–49?
e) was 30 and under, given that they bought juice?

| Age | Soda | Juice | Total |
|---|---|---|---|
| 30 and under | 14 | 32 | 46 |
| 31–49 | 23 | 22 | 45 |
| 50 and over | 17 | 13 | 30 |
| Total | 54 | 67 | 121 |

a) $\frac{54}{121} = 0.446281$  b) $\frac{30}{121} = 0.247934$  c) $\frac{23}{121} = 0.190083$

d) $\frac{23}{45} = 0.511111$

e) $\frac{32}{67} = 0.477612$

**4.** (10pts) A company wishes to offer insurance against failure of the Sangfroid smartphone during its first two years of use. It figures that 5% of Sangfroids fail during the first year, and 12% fail during the second year. An insured customer receives $100 if their Sangfroid fails during the first year, and $75 if it fails during the second year.
a) What is the expected payout of a policy?
b) What premium should the company charge in order to break even?
c) What premium should the company charge in order to profit $20 per policy?

a)
| outcome (payout) | probability |
|---|---|
| 100 | 0.05 |
| 75 | 0.12 |
| 0 | 0.83 |

b) $14 (to cover payout)

c) $34 = 14 + 20
       payout   profit

expected value $= 100\cdot 0.05 + 75\cdot 0.12 + 0\cdot 0.83$
$= 5 + 9 + 0 = \$14$ expected payout per policy

**5.** (14pts) A game of chance is set up as follows: A player pays $3 and two dice are rolled. The player collects $10 if the sum on the dice is 2 or 12, $5 if the sum is 9, $2 if sum is 6 and nothing if the sum is any other number.
a) Find the expected value of this game.
b) If you play this game 20 times, how much do you expect to win or lose?
c) What is the fair price of this game?

| outcome | probability |
|---|---|
| 10-3 = 7 | $\frac{2}{36}$ |
| 5-3 = 2 | $\frac{4}{36}$ |
| 2-3 = -1 | $\frac{5}{36}$ |
| -3 | $\frac{25}{36}$ ← 36-11 |

2 or 12: 1,1; 6,6
9: 3,6; 4,5; 5,4; 6,3
6: 1,5; 2,4; 3,3; 4,2; 5,1

Expected value $= 7\cdot\frac{2}{36} + 2\cdot\frac{4}{36} + (-1)\frac{5}{36} + (-3)\cdot\frac{25}{36}$
$= \frac{14+8-5-75}{36} = -\frac{58}{36} = -\frac{29}{18} = -1.61$

b) $-1.61\cdot 20 = -\$32.20$

c) $-1.61 + 3 = 1.39$