Mathematical Concepts — Joysheet 3
MAT 117, Spring 2012 — D. Ivanšić

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1. (8pts) DJ Phonick would like to have $2000 for a good turntable system. How much should he deposit now in an account bearing 3.75%, compounded quarterly in order to have the desired amount in one and a half years? How much of the $2000 came from interest?

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$2000 = P\left(1+\frac{0.0375}{4}\right)^{4\cdot 1.5}$

$2000 = P\left(1.009375\right)^{6}$

$2000 = P \cdot 1.05\ldots$

$P = \frac{2000}{1.05\ldots} = 1891.10$

From interest:

$2000 - 1891.10 = 108.90$

2. (6pts) Bank of Eggner's Ferry is offering a 2.45% interest rate on an account that is compounded quarterly, while New Bridge Bank has an account at 2.35%, compounded daily. Which account is the better deal?

$Y = \left(1+\frac{r}{n}\right)^{n} - 1$

Eggner's Ferry:

$\left(1+\frac{0.0245}{4}\right)^{4} - 1$

$= 0.024726\ldots$

$Y = 2.472601\%$ ← better deal

New Bridge Bank:

$\left(1+\frac{0.0235}{365}\right)^{365} - 1$

$= 0.02377\ldots$

$Y = 2.377753\%$

3. (10pts) If you deposit $1000 into an account bearing 7% interest, compounded monthly, how long will it take until you have $3500 in the account?

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$3500 = 1000\left(1+\frac{0.07}{12}\right)^{12t} \mid \div 1000$

$3.5 = \left(1.00583\ldots\right)^{12t} \mid \log$

$\log 3.5 = \log\left(1.00583\ldots\right)^{12t}$

$\log 3.5 = 12t \log\left(1.00583\ldots\right) \mid \div 12\log\left(1.00583\ldots\right)$

$t = \frac{\log 3.5}{12\log(1.00583\ldots)}$

$= 17.948762$

About 17.95 years

4. (10pts) To save for an addition to the house, the Jorgensons deposit \$300 every month into an account bearing 3.25% interest, compounded monthly.
a) How much do they have in the account in 4 years?
b) How much did they earn in interest over these 4 years?

a) $A = P\dfrac{\left(1+\frac{r}{n}\right)^{nt}-1}{\frac{r}{n}}$

$A = 300\dfrac{\left(1+\frac{0.0325}{12}\right)^{12\cdot 4}-1}{\frac{0.0325}{12}}$

$A = 300 \cdot 51.18\ldots = 15{,}355.75$

b) interest = A − (total deposits)

$= 15{,}355.75 - 300 \cdot 48$

$= 955.75$

5. (14pts) At the time of little Mia's birth, her parents decided to save some money for her college. They set aside \$1000 every quarter for 12 years in an account bearing 4.5% interest, compounded quarterly. Then, financial hardship forced them to stop contributing, but they left the accumulated money in the account.
a) How much is in the account when Mia is 18?
b) How much of it was from deposits, and how much from interest?

a) After 12 years, amt. in account is

$A = 1000 \cdot \dfrac{\left(1+\frac{0.045}{4}\right)^{4\cdot 12}-1}{\frac{0.045}{4}}$

$A = 1000 \cdot 63.185\ldots$

$= 63{,}185.87$

The next six years no additional deposits are made, so it's like a one-time deposit:

$A = 63{,}185.87 \cdot \left(1+\frac{0.045}{4}\right)^{4\cdot 6}$

$= 63{,}185.87 \cdot 1.30795$

$= 82{,}646.56$

b) From deposits: $12 \cdot 4 \cdot 1000 = 48000$

From interest: $82{,}646.56 - 48000$

$= 34{,}646.56$

6. (12pts) The Jimenezes would like to save \$20,000 for a new car. If they can set aside \$300 every month into an account bearing 3.72%, compounded monthly, how long will it take them to save the desired amount?

$A = P\dfrac{\left(1+\frac{r}{n}\right)^{nt}-1}{\frac{r}{n}}$

$20000 = 300\dfrac{\left(1+\frac{0.0372}{12}\right)^{12t}-1}{\frac{0.0372}{12}} \quad |\div 300$

$66.66\ldots = \dfrac{(1.0031)^{12t}-1}{0.0031} \quad |\cdot 0.0031$

$0.2066\ldots = (1.0031)^{12t}-1 \quad |+1$

$1.2066\ldots = (1.0031)^{12t} \quad |\log$

$\log(1.2066\ldots) = 12t\log(1.0031) \quad |\div 12\log 1.0031$

$t = \dfrac{\log(1.2066\ldots)}{12\log(1.0031)} = 5.05787$

About 5 years.