

**Mathematical Concepts — Exam 1**  
**MAT 117, Spring 2012 — D. Ivanšić**

**Name:** \_\_\_\_\_  
*Show all your work!*

$$I = Prt \quad A = P(1 + rt) \quad A = P \left(1 + \frac{r}{n}\right)^{nt} \quad A = P \frac{\left(1 + \frac{r}{n}\right)^{nt} - 1}{\frac{r}{n}} \quad P = PMT \frac{1 - \left(1 + \frac{r}{n}\right)^{-nt}}{\frac{r}{n}} \quad Y = \left(1 + \frac{r}{n}\right)^n - 1$$

1. (10pts) a) 12 is what percent of 25?

b) 7 is 64% of what?

2. (10pts) You borrowed \$400 from a friend, and repaid him in 5 months with \$450. What simple annual interest rate did you pay on this loan?

3. (8pts) How much should you deposit in an account bearing 3.47%, compounded semi-annually, if you would like to have \$4,000 in three years?

4. (14pts) In 2010, single man Fidelio filed income taxes His total income was \$85,300, he deposited \$5000 into a retirement account, paid \$7400 in mortgage interest, \$2100 in property taxes, \$3900 in state income taxes and donated \$750 to charity. Use the table below to first determine Fidelio's taxable income (don't forget the exemption) and then find the tax on this income.

Income bracket	Tax rate
up to \$8,375	10%
\$8,376-\$34,000	15%
\$34,001-\$82,400	25%
\$82,401-\$171,850	28%
\$171,851-\$373,650	33%
more than \$373,650	35%
exemption	\$3650

5. (14pts) You would like to save up for a nice new computer.

a) How much should you deposit every day into an account with 3.5% interest, compounded daily, in order to have \$2,500 in three years?

b) How much of the final amount is from deposits and how much from interest?

**6.** (32pts) True story: in an attempt at reducing Demi's anger at his indiscretions, Ashton bought her a very nice car for \$103,000. Made-up part: for this he took on a 7-year loan at 5.25%, compounded monthly.

a) What is his monthly payment on the loan?

b) What are his total payments over the course of the loan? How much of this amount is for interest?

c) How much of his first payment goes toward interest, and how much towards the principal?

d) How much does he owe after 3 years?

7. (12pts) If you deposit \$2000 into an account bearing 11% interest, compounded quarterly, how long will it take until you have \$6000 in the account?

**Bonus.** (10pts) In an effort to save for a down payment on a home, Maria deposits \$3000 into an account, bearing 4.5%, compounded monthly. Her financial situation does not allow her to make any additional deposits for a year. Then, after getting a promotion at work, she starts making monthly deposits of \$200 for the next two years into the same account. How much is in the account in three years from now?

**Mathematical Concepts — Exam 2**  
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$$\frac{a}{b} = \frac{(E)}{1-P(E)} \quad P(E) = \frac{a}{a+b} \text{ where odds in favor of } E \text{ are } a : b \quad P(B | A) = \frac{n(A \text{ and } B)}{n(A)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \text{ or } B) = P(A) + P(B) \text{ (if } A \text{ and } B \text{ are mutually exclusive)}$$

$$P(A \text{ and } B) = P(A) \cdot P(B | A) \quad P(A \text{ and } B) = P(A) \cdot P(B) \text{ if } A \text{ and } B \text{ are independent}$$

$$E = P_1 \cdot A_1 + P_2 \cdot A_2 + \cdots + P_n \cdot A_n$$

1. (6pts) In a restaurant, there are 17 choices for appetizers, 43 for the main dish and 13 for dessert. Given these choices, how many different three-course meals could you have?

2. (6pts) A die is rolled four times. How many different outcomes does this experiment have?

3. (14pts) The table shows the pattern of mammography results and breast cancer rates among a number of U.S. women ages 40–50. Assuming the numbers are representative for the general population, what is the probability that a random U.S. woman between the ages of 40 and 50:

- has breast cancer?
- had a positive mammogram?
- does not have breast cancer and had a positive mammogram?
- has breast cancer, given her mammogram is positive?
- doesn't have breast cancer, given her mammogram is negative?
- has a negative mammogram, given that she has breast cancer?

	Cancer	No Cancer	Total
Positive mammo.	720	6,944	
Negative mammo.	80	92,256	
Total			

4. (18pts) Write the probabilities and odds against and in favor of the following events (you can show any work needed below):

Event	probability	odds against	odds in favor
a) Rolling a 4 on a single roll of a die			
b) Drawing a red face card from a deck of cards			
c) Getting at least one tail on three coin tosses			
d) Getting sum 7 or 8 on a roll of two dice			
e) Both numbers odd on a roll of two dice			

5. (14pts) A spinner has 8 equal-size fields, one of which is labeled W, two are labeled I and five are labeled N. A game of chance is set up like this: the player pays \$5 and spins. Depending on whether the spinner lands on W, I or N the player wins \$15, \$7 or nothing, respectively.

- Find the expected value of this game.
- What is the fair price of this game?
- If a player played this game 100 times, how much would they expect to win or lose?

6. (10pts) In the ice cream bin of a convenience store, 66% of the products contain vanilla, 47% contain chocolate, and 31% have both of those ingredients. If an ice cream product is selected at random, what is the probability that:

- a) it contains vanilla or chocolate?
- b) it lacks at least one of those ingredients?

7. (14pts) A picky music lover browsing through an online music store finds that, in his opinion, 65% of the tracks there suck. Choosing tracks randomly, what is the probability that he will select

- a) on two tries, both tracks that don't suck?
- b) on three tries, all three tracks that suck?
- c) on four tries, at least one track that doesn't suck?

8. (18pts) An animal shelter has 7 black kittens, 4 calicos and 5 grey kittens. If you pick two kitties at random, what is the probability that:

a) both are calicos?

b) the first is black and the second is gray?

c) exactly one is grey?

**Bonus.** (10pts) Two cards are drawn from a deck at random. What is the probability that the first one is a face card and the second one is a club? *Hint: this will help you somewhere in the problem: if  $B$  and  $C$  are mutually exclusive,  $P(B \text{ or } C | A) = P(B | A) + P(C | A)$ .*



$$\begin{aligned} \text{midrange} &= \frac{\text{lowest value} + \text{highest value}}{2} & \text{range} &= \text{highest value} - \text{lowest value} \\ \bar{x} &= \frac{x_1 + x_2 + \cdots + x_n}{n} = \frac{\sum_i x_i}{n} = \frac{\sum_i x_i f_i}{n} & Z &= \frac{X - \bar{x}}{s} \\ s &= \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2}{n - 1}} = \sqrt{\frac{\sum_i (x_i - \bar{x})^2}{n - 1}} = \sqrt{\frac{\sum_i f_i (x_i - \bar{x})^2}{n - 1}} \end{aligned}$$

1. (18pts) Over the course of two weeks (workdays only) DMV employee Jonathan counts the daily number of people coming to take the driving test. He gets the following numbers: 8, 7, 4, 11, 10, 9, 9, 10, 7, 6.

- Find the midrange.
- Find the median.
- Find the mean.
- Find the range.
- Find the standard deviation.

2. (8pts) A downtown beautification project is proposed for Murray's downtown, paid for by taxpayer money. To gauge support for the idea, city officials decide to do a survey of the city's population. Comment on whether each of the following methods will produce a good random sample of the city's population:

- Surveying passers-by on the court square.
- Surveying people who have come to see a basketball game at the CFSB center.
- Picking random names from the list of people who pay property taxes in Murray, and surveying them.
- Surveying patrons of Murray's Backyard Burger.

**3.** (25pts) A fashion magazine assistant editor does not want to repeat the clothes she wears too often and has her assistant keep track of how often she wore each of her garments over a period of time. The table below indicates how many times she wore those garments (she wore 18 garments once, 14 garments twice, etc.). Note that “times worn” is the data, the other numbers are frequencies.

- a) Draw a histogram for the data.
- b) Find the mode number of times worn.
- c) Find the median number of times worn.
- d) Find the mean number of times worn.
- e) Find the standard deviation.

Times worn	Frequency (garments)
1	18
2	14
3	10
4	17
5	6

**4.** (6pts) In a normal distribution with mean 15 and standard deviation 2.1, which data value is

- a) 2 standard deviations above the mean?
- b) 1.5 standard deviations below the mean?

5. (14pts) The scores on exam 2 of a College Algebra class are shown below.

a) Construct a grouped frequency distribution whose first class is 50–59.

b) Enter a representative value for each interval.

c) Use the representative values to estimate the mean of data. How does it compare to the actual mean of 79.8? (Do not compute the actual mean.)

91, 109, 83, 92, 68, 75, 74, 86, 84, 69, 83, 65, 92, 69, 76, 100, 89, 74, 53, 64, 78, 67, 73, 101, 78, 72, 58, 74, 96, 59, 102, 71, 85, 64, 77, 96, 107

Range of Scores	Frequency	Representative Value

6. (12pts) Assume the speed of vehicles on highway 641 in the afternoon is normally distributed with mean of 59mph and standard deviation 4mph. Use the 68-95-99.7 rule (draw a picture) to find the percentage of drivers whose speed is:

a) between 59 and 63

b) under 51

c) over 63

d) between 55 and 67

7. (17pts) Systolic blood pressure readings are normally distributed with a mean of 121 and a standard deviation of 15. Draw a picture showing which area you are computing as you answer:

- a) What percentage of people have systolic blood pressure between 90 and 110?
- b) What percentage of people have systolic blood pressure higher than 140 (which is considered high blood pressure)?

**Bonus.** (10pts) A woman insists she will never marry a man shorter than she is, even though she knows only 20% of men fall into this category. If men's heights are normally distributed with mean 69 inches and standard deviation 2.5 inches, how tall is the woman? (*Hint: this problem is the reverse of what we usually do with a normal distribution. Here, the area is given: you have to find the  $z$ -score that this area corresponds to. You will also need to find the height corresponding to that  $z$ -score.*)

**Mathematical Concepts — Exam 4**  
**MAT 117, Spring 2012 — D. Ivanšić**

**Name:** \_\_\_\_\_  
*Show all your work!*

1. (24pts) A class is planning to celebrate the end of the semester at a Murray bar. To decide which one to choose, the students take a vote between The Big Apple (A), El Mariachi (M) and 15th&Olive (O).

Votes:	6	3	2	4	1	7
1st	A	A	M	M	O	O
2nd	M	O	A	O	A	M
3rd	O	M	O	A	M	A

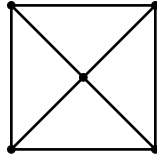
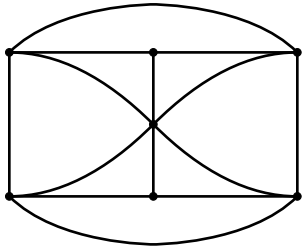
- Which choice wins the vote in a plurality election?
- Which choice wins the vote in a plurality election with a runoff?
- Which choice is the pairwise comparison winner?
- Which choice is the winner using Borda's method? Perform the check on the sum of Borda points.

2. (30pts) At an astronomers' congress, the participants are deciding to which planet to send the next planetary exploration spacecraft. The target is one of the not so well explored planets: Mercury, Neptune, Uranus, or Venus. The preference rankings of the astronomers broke down into the following percentages.

Votes	14	13	12	16	15	20	10
1st	M	N	N	U	U	V	V
2nd	V	U	U	N	V	M	U
3rd	N	V	M	V	N	U	N
4th	U	M	V	M	M	N	M

- Which choice wins the vote in a plurality election?
- Which choice wins the vote in a plurality election with a runoff?
- Which choice is the pairwise comparison winner?
- Which choice is the winner using Borda's method? Perform the check on the sum of Borda points.

3. (12pts) Determine whether each of the following graphs has an Euler path or an Euler circuit. If it does, find it, if not, explain why not.

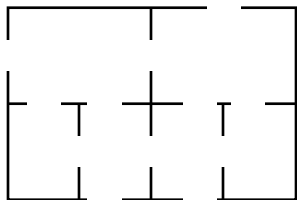


4. (14pts) Below is a floor plan of a section of a museum, with doors joining rooms indicated.

a) Represent the floor plan as a graph (rooms are vertices, don't forget to include an "outside").

b) Use the graph to determine if it is possible to walk around the section, passing through every door exactly once. If it is, draw the route.

c) Is it possible to do the same as in b), and start and finish outside?



5. (20pts) A tourist would like to visit Los Angeles, Las Vegas, Phoenix and San Francisco while trying to minimize the distance traveled. The table below has the distances between the cities.

a) Draw a weighted graph that corresponds to this problem.

b) Use the brute force method to find the route that minimizes the distance traveled. First list all the possible orders of visits with Las Vegas as the starting city.

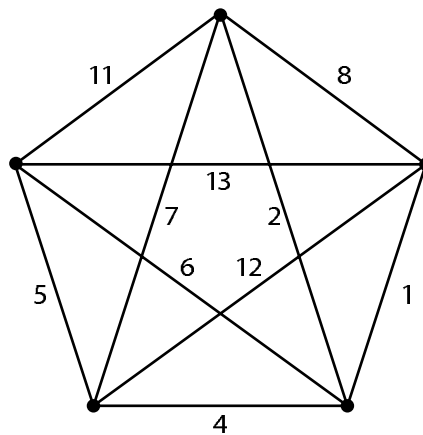
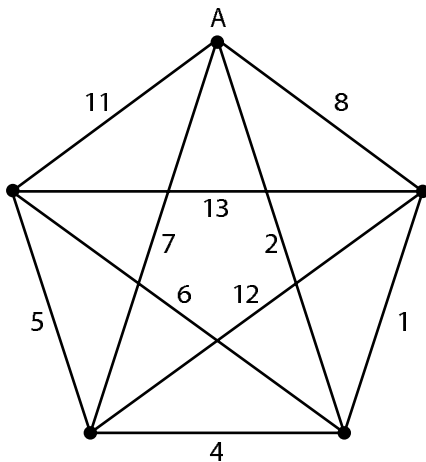
c) Use the nearest neighbor algorithm to find an approximate solution to the problem. Is it the same as in c)?

	LV	LA	P
LA	272		
P	292	372	
SF	579	381	750

**Bonus.** (10pts) Find an approximate solution to the traveling salesman problem. Show the weight of the found circuits. Use (one on each picture)

a) the nearest neighbor algorithm starting at A.

b) the greedy algorithm.





$$I = Prt \quad A = P(1 + rt) \quad A = P \left(1 + \frac{r}{n}\right)^{nt} \quad A = P \frac{\left(1 + \frac{r}{n}\right)^{nt} - 1}{\frac{r}{n}}$$

$$P = PMT \frac{1 - \left(1 + \frac{r}{n}\right)^{-nt}}{\frac{r}{n}} \quad Y = \left(1 + \frac{r}{n}\right)^n - 1$$

$$\frac{a}{b} = \frac{P(E)}{1 - P(E)} \quad P(E) = \frac{a}{a+b} \text{ where odds in favor of } E \text{ are } a : b \quad P(B | A) = \frac{n(A \text{ and } B)}{n(A)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \text{ or } B) = P(A) + P(B) \text{ (if } A \text{ and } B \text{ are mutually exclusive)}$$

$$P(A \text{ and } B) = P(A) \cdot P(B | A) \quad P(A \text{ and } B) = P(A) \cdot P(B) \text{ if } A \text{ and } B \text{ are independent}$$

$$E = P_1 \cdot A_1 + P_2 \cdot A_2 + \dots + P_n \cdot A_n$$

$$\text{midrange} = \frac{\text{lowest value} + \text{highest value}}{2} \quad \text{range} = \text{highest value} - \text{lowest value}$$

$$\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n} = \frac{\sum_i x_i}{n} = \frac{\sum_i x_i f_i}{n} \quad Z = \frac{X - \bar{x}}{s}$$

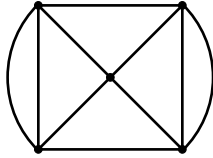
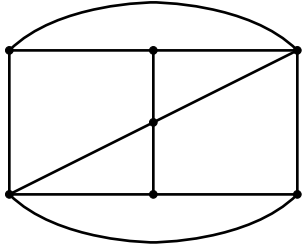
$$s = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \dots + (x_n - \bar{x})^2}{n - 1}} = \sqrt{\frac{\sum_i (x_i - \bar{x})^2}{n - 1}} = \sqrt{\frac{\sum_i f_i (x_i - \bar{x})^2}{n - 1}}$$

1. (24pts) Financial backers of the outdoor artist Christo are deciding in which world city his newest installation is to appear. They rank the three possible choices, Chicago, Kuala Lumpur and Naples in order of preference

- Which choice wins the vote in a plurality election?
  - Which choice wins the vote in a plurality election with elimination?
  - Which choice is the pairwise comparison winner?
  - Which choice is the winner using Borda's method?
- Perform the check on the sum of Borda points.

Votes:	4	2	5	3	7	2
1st	C	C	K	K	N	N
2nd	K	N	C	N	C	K
3rd	N	K	N	C	K	C

2. (12pts) Determine whether each of the following graphs has an Euler path or an Euler circuit. If it does, find it, if not, explain why not.



3. (16pts) Systolic blood pressure readings are normally distributed with a mean of 121 and a standard deviation of 15. Draw a picture showing which area you are computing as you answer:

- What percentage of people have systolic blood pressure between 90 and 110?
- What percentage of people have systolic blood pressure higher than 140 (which is considered high blood pressure)?

4. (24pts) A hospital keeps track of the number of days a patient stays in the hospital for an appendectomy. Below is the data.

- a) Draw a histogram for the data.
- b) Find the mode number of days spent.
- c) Find the median number of days spent.
- d) Find the mean number of days spent.
- e) Find the standard deviation.

Days spent	Frequency (patients)
2	13
3	17
4	25
5	11
6	4

5. (12pts) Write the probabilities and odds against and in favor of the following events (you can show any work needed below):

Event	probability	odds against	odds in favor
a) Rolling a 3 on a single roll of a die			
b) Drawing a king or an ace from a deck of cards			
d) Getting sum 3 or 10 on a roll of two dice			

6. (6pts) The parking lot of a car dealership has 80 cars, of which 26 are used, 18 are silver, and 5 are used and silver. If a car is selected from the lot at random, what is the probability that it is used or silver?

7. (10pts) The probability that it will snow on Christmas in a certain Minnesota town is 45%. Assume that whether it snows on Christmas in one year is independent of whether it snows on Christmas in any other year. What is the probability that it

a) snows on two Christmases in a row?

b) snows at least once on three Christmases in a row?

8. (12pts) An enterprising student devised this game of chance with his classmates in Dr. Smirf's class: a player pays \$4 to play and receives a prize if they guess the color Dr. Smirf's shirt the next day. The player wins \$10 if Dr. Smirf's shirt is red, \$6 if it is green, \$3 if it is black and nothing in all other cases. Suppose Dr. Smirf wears a red shirt on 10% of days, a green shirt on 20% of days, a black shirt on 30% of days, and a shirt of another color on other days.

a) Find the expected value of this game.

b) If a player played this game 30 times, how much would they expect to win or lose?

**9.** (6pts) Kristina borrowed \$1,000 from the bank at a 7% simple annual rate, and repaid the loan in 9 months. How much did she pay back?

**10.** (14pts) You would like to save up for a car.

a) How much should you deposit every quarter into an account with 5% interest, compounded quarterly, in order to have \$15,000 in four years?

b) How much of the final amount is from deposits and how much from interest?

**11.** (14pts) French president Sarkozy just lost his reelection bid. In an effort to console himself, he purchases a yacht for 1 million euros and takes out a 10-year loan for that amount at 6.6%, compounded monthly.

a) What is his monthly payment on the loan?

b) What are his total payments over the course of the loan? How much of this amount is for interest?

**Bonus.** (10pts) Below is a floor plan of a section of a museum, with doors joining rooms indicated.

a) Represent the floor plan as a graph (rooms are vertices, and don't forget to include one vertex for "outside").

b) Use the graph to determine if it is possible to walk around the section, passing through every door exactly once. If it is, draw the route.

c) Is it possible to do the same as in b), and start and finish outside?

