Mathematical Concepts — Final Exam
MAT 117, Spring 2012 — D. Ivanšić

Name: Saul Ocean

Show all your work!

$I = Prt$ $A = P(1+rt)$ $A = P\left(1+\frac{r}{n}\right)^{nt}$ $A = P\frac{\left(1+\frac{r}{n}\right)^{nt}-1}{\frac{r}{n}}$

$P = PMT\frac{1-\left(1+\frac{r}{n}\right)^{-nt}}{\frac{r}{n}}$ $Y = \left(1+\frac{r}{n}\right)^{n}-1$

$\frac{a}{b} = \frac{(E)}{1-P(E)}$ $P(E) = \frac{a}{a+b}$ where odds in favor of E are $a:b$ $P(B\,|\,A) = \frac{n(A \text{ and } B)}{n(A)}$

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$P(A \text{ or } B) = P(A) + P(B)$ (if A and B are mutually exclusive)

$P(A \text{ and } B) = P(A)\cdot P(B\,|\,A)$ $P(A \text{ and } B) = P(A)\cdot P(B)$ if A and B are independent

$E = P_1\cdot A_1 + P_2\cdot A_2 + \cdots + P_n\cdot A_n$

$\text{midrange} = \frac{\text{lowest value} + \text{highest value}}{2}$ $\text{range} = \text{highest value} - \text{lowest value}$

$\overline{x} = \frac{x_1+x_2+\cdots+x_n}{n} = \frac{\sum_i x_i}{n} = \frac{\sum_i x_i f_i}{n}$ $Z = \frac{X-\overline{x}}{s}$

$s = \sqrt{\frac{(x_1-\overline{x})^2+(x_2-\overline{x})^2+\cdots+(x_n-\overline{x})^2}{n-1}} = \sqrt{\frac{\sum_i (x_i-\overline{x})^2}{n-1}} = \sqrt{\frac{\sum_i f_i(x_i-\overline{x})^2}{n-1}}$

1. (24pts) Financial backers of the outdoor artist Christo are deciding in which world city his newest installation is to appear. They rank the three possible choices, Chicago, Kuala Lumpur and Naples in order of preference

a) Which choice wins the vote in a plurality election?
b) Which choice wins the vote in a plurality election with elimination?
c) Which choice is the pairwise comparison winner?
d) Which choice is the winner using Borda's method? Perform the check on the sum of Borda points.

Votes:	4	2	5	3	7	2
1st	C	C	K	K	N	N
2nd	K	N	C	N	C	K
3rd	N	K	N	C	K	C

$= 23$

a) C 6
K 8
N 9 wins

b) K, N advance to 2nd round
K 8+4 = 12 wins
N 9+2 = 11

c) C 6+7 = 13 wins
K 8+2 = 10

C 6+5 = 11
N 9+3 = 12 wins

K 8+4 = 12 wins
N 9+2 = 11

	C	K	N
pts.	1	1	1

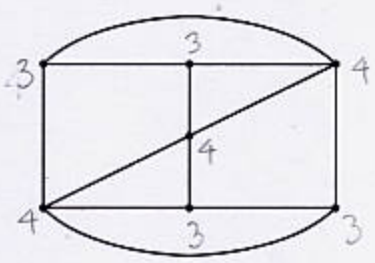
It's a tie

d) C $6\cdot3 + 12\cdot2 + 5\cdot1 = 47$ wins
K $8\cdot3 + 6\cdot2 + 9\cdot1 = 45$
N $9\cdot3 + 5\cdot2 + 9\cdot1 = 46$

13.8

$23\cdot6 = 138$, checks out,
↑ pts. per ballot

2. (12pts) Determine whether each of the following graphs has an Euler path or an Euler circuit. If it does, find it, if not, explain why not.

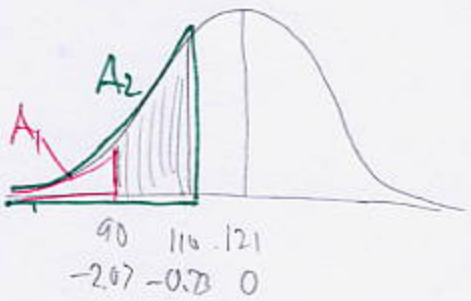


Has more than two odd vertices, hence no Euler path nor circuit.

Has no odd vertices, hence has an Euler path.

3. (16pts) Systolic blood pressure readings are normally distributed with a mean of 121 and a standard deviation of 15. Draw a picture showing which area you are computing as you answer:
a) What percentage of people have systolic blood pressure between 90 and 110?
b) What percentage of people have systolic blood pressure higher than 140 (which is considered high blood pressure)?

a)



$$z = \frac{110-121}{15} \approx -0.73$$

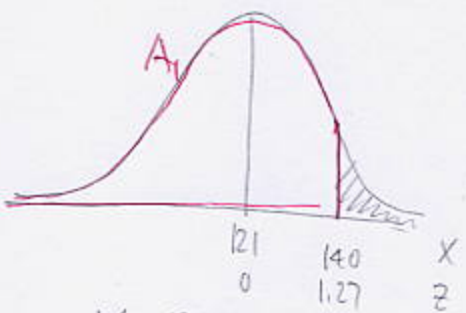
$$z = \frac{90-121}{15} \approx -2.07$$

$$A(-2.07 \le z \le -0.73) = A_2 - A_1$$

$$= 0.2327 - 0.0192$$

$$= 0.2135 \qquad 21.35\%$$

b)



$$z = \frac{140-121}{15} \approx 1.27$$

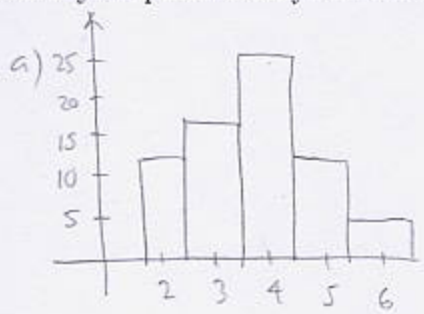
$$A(z \ge 1.27) = 1 - A_1$$

$$= 1 - 0.8980 = 0.1020$$

$$10.2\%$$

4. (24pts) A hospital keeps track of the number of days a patient stays in the hospital for an appendectomy. Below is the data.

a) Draw a histogram for the data.
b) Find the mode number of days spent.
c) Find the median number of days spent.
d) Find the mean number of days spent.
e) Find the standard deviation.



Days spent	Frequency (patients)
2	13
3	17
4	25
5	11
6	4
	70

b) mode = 4

c) $\frac{70}{2} = 35$, need mean of 35th, 36th data item.

2, 2, 3, 3, 4, 4
↑ 13th, ↑ 30th, ↑ 55th

35th and 36th are 4

median $= \frac{4+4}{2} = 4$

d) $\bar{x} = \frac{13\cdot 2 + 17\cdot 3 + 25\cdot 4 + 11\cdot 5 + 4\cdot 6}{13+17+25+11+4} = \frac{242}{70} = 3.457143$

e) $13(2-3.45\_)^2 + 17(3-3.45\_)^2 + \ldots + 4(6-3.45\_)^2 = 90.5714\ldots$

$s = \sqrt{\frac{90.571\_}{70-1}} = 1.145701$

5. (12pts) Write the probabilities and odds against and in favor of the following events (you can show any work needed below):

	Event	probability	odds against	odds in favor
a)	Rolling a 3 on a single roll of a die	$\frac{1}{6}$	5 : 1	1 : 5
b)	Drawing a king or an ace from a deck of cards	$\frac{8}{52} = \frac{2}{13}$	44:8 = 11 : 2	2 : 11
d)	Getting sum 3 or 10 on a roll of two dice	$\frac{5}{36}$	31 : 5	5 : 31

b) 4 kings, 4 aces among 52 cards

$\frac{8}{52}$

c) sum 3: 1,2 / 2,1 — sum 10: 4,6 / 5,5 / 6,4

6. (6pts) The parking lot of a car dealership has 80 cars, of which 26 are used, 18 are silver, and 5 are used and silver. If a car is selected from the lot at random, what is the probability that it is used or silver?

$P(\text{used or silver}) = P(\text{used}) + P(\text{silver}) - P(\text{used and silver})$

$= \frac{26}{80} + \frac{18}{80} - \frac{5}{80} = \frac{39}{80}$

7. (10pts) The probability that it will snow on Christmas in a certain Minnesota town is 45%. Assume that whether it snows on Christmas in one year is independent of whether it snows on Christmas in any other year. What is the probability that it

a) snows on two Christmases in a row?

b) snows at least once on three Christmases in a row?

a) $P(\text{snows on 1st and snows on 2nd}) = P(\text{snows on 1st}) \cdot P(\text{snows on 2nd}) = 0.45 \cdot 0.45 = 0.2025$

b) $P(\text{snows at least once}) = 1 - P(\text{doesn't snow three times in a row})$

$= 1 - P(\text{doesn't snow on 1st}) \cdot P(\text{doesn't snow on 2nd}) \cdot P(\text{doesn't snow on 3rd}) =$

$= 1 - 0.55 \cdot 0.55 \cdot 0.55 = 1 - 0.55^3 = 0.833625$

8. (12pts) An enterprising student devised this game of chance with his classmates in Dr. Smirf's class: a player pays \$4 to play and receives a prize if they guess the color Dr. Smirf's shirt the next day. The player wins \$10 if Dr. Smirf's shirt is red, \$6 if it is green, \$3 if it is black and nothing in all other cases. Suppose Dr. Smirf wears a red shirt on 10% of days, a green shirt on 20% of days, a black shirt on 30% of days, and a shirt of another color on other days.

a) Find the expected value of this game.

b) If a player played this game 30 times, how much would they expect to win or lose?

a)

Net financial outcome	probability
$10-4=6$	0.10
$6-4=2$	0.20
$3-4=-1$	0.30
-4	0.40

$E = 6 \cdot 0.1 + 2 \cdot 0.2 + (-1) \cdot 0.3 + (-4) \cdot 0.4$

$= -0.9$ player loses 90¢ per game, on average.

b) $0.9 \cdot 30 = 27$

Expect to lose \$27

9. (6pts) Kristina borrowed \$1,000 from the bank at a 7% simple annual rate, and repaid the loan in 9 months. How much did she pay back?

$A = P(1+rt)$

$A = 1000\left(1+0.07\cdot\frac{9}{12}\right) = 1000\left(1+0.07\cdot\frac{3}{4}\right) = 1000\cdot 1.0525 = 1052.50$

10. (14pts) You would like to save up for a car.
a) How much should you deposit every quarter into an account with 5% interest, compounded quarterly, in order to have \$15,000 in four years?
b) How much of the final amount is from deposits and how much from interest?

a) $A = P\,\dfrac{\left(1+\frac{r}{n}\right)^{nt}-1}{\frac{r}{n}}$

$15{,}000 = P\,\dfrac{\left(1+\frac{0.05}{4}\right)^{4\cdot 4}-1}{\frac{0.05}{4}}$

$15{,}000 = P\cdot\dfrac{1.0125^{16}-1}{0.0125}$

$15{,}000 = P\cdot 17.59\_\_$

$P = \dfrac{15000}{17.59\_} = 852.70$

b) $852.70\cdot 16 = 13{,}643.20$ from principal

$15000 - 13{,}643.20 = 1356.80$ From interest

11. (14pts) French president Sarkozy just lost his reelection bid. In an effort to console himself, he purchases a yacht for 1 million euros and takes out a 10-year loan for that amount at 6.6%, compounded monthly.

a) What is his montly payment on the loan?

b) What are his total payments over the course of the loan? How much of this amount is for interest?

a) $A = PMT \dfrac{1-\left(1+\frac{r}{n}\right)^{-nt}}{\frac{r}{n}}$

$1{,}000{,}000 = PMT \dfrac{1-\left(1+\frac{0.66}{12}\right)^{-12\cdot 10}}{\frac{0.66}{12}}$

$1{,}000{,}000 = PMT \dfrac{1-(1.0055)^{-120}}{0.0055}$

$1{,}000{,}000 = PMT \cdot 87.67$

$PMT = \dfrac{1{,}000{,}000}{87.67} = €11{,}405.74$

↑ monthly payment

b) Total payments $= 11{,}405.74 \cdot 120$

$= 1{,}368{,}688.80$

For interest, it is

$1{,}368{,}688.80 - 1{,}000{,}000$

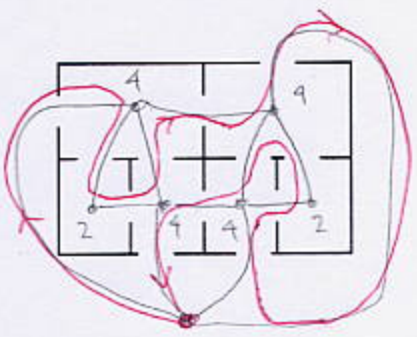
$= 368{,}688.80$

Bonus. (10pts) Below is a floor plan of a section of a museum, with doors joining rooms indicated.

a) Represent the floor plan as a graph (rooms are vertices, and don't forget to include one vertex for "outside").

b) Use the graph to determine if it is it possible to walk around the section, passing through every door exactly once. If it is, draw the route.

c) Is it possible to do the same as in b), and start and finish outside?



b) All vertices are even, so a circuit exists

c) yes, because a circuit may start and finish at any part.