Mathematical Concepts — Exam 4
MAT 117, Spring 2012 — D. Ivanšić

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Show all your work!

1. (24pts) A class is planning to celebrate the end of the semester at a Murray bar. To decide which one to choose, the students take a vote between The Big Apple (A), El Mariachi (M) and 15th&Olive (O).

Votes:	6	3	2	4	1	7
1st	A	A	M	M	O	O
2nd	M	O	A	O	A	M
3rd	O	M	O	A	M	A

→ 23 voters

a) Which choice wins the vote in a plurality election?
b) Which choice wins the vote in a plurality election with a runoff?
c) Which choice is the pairwise comparison winner?
d) Which choice is the winner using Borda's method? Perform the check on the sum of Borda points.

a) A: 6+3=9 wins
M: 2+4=6
O: 1+7=8

b) Runoff is between A and O
A: 9+2=11
O: 8+4=12 wins

c) A: 9+1=10
M: 6+7=13 wins

A: 9+2=11
O: 8+4=12 wins

M: 6+6=12 wins
O: 8+3=11

Points:
A 1
M 2 wins
O 0

d) Borda points:
A: 9·3+3·2+11·1=44
M: 6·3+13·2+4·1=48 wins
O: 8·3+7·2+8·1=46
138

6·23=138, so it checks out.

2. (30pts) At an astronomers' congress, the participants are deciding to which planet to send the next planetary exploration spacecraft. The target is one of the not so well explored planets: Mercury, Neptune, Uranus, or Venus. The preference rankings of the astronomers broke down into the following percentages.

total = 100% of votes

Votes	14	13	12	16	15	20	10
1st	M	N	N	U	U	V	V
2nd	V	U	U	N	V	M	U
3rd	N	V	M	V	N	U	N
4th	U	M	V	M	M	N	M

a) Which choice wins the vote in a plurality election?
b) Which choice wins the vote in a plurality election with a runoff?
c) Which choice is the pairwise comparison winner?
d) Which choice is the winner using Borda's method? Perform the check on the sum of Borda points.

a) M 14
N 13+12=25
U 16+15=31 wins
V 20+10=30

b) Second round:
N 25 ← deleted
U 31
V 30+14=44

Third round:
U 31+13+12=56 wins
V 44 =44

c) M 14+12 = 26 V 30+14+15=59 wins
V 30+13+16+15=74 wins N 25+16 =41

M 14+20 = 34 V 30+14 =44
N 25+16+15+10=66 wins U 31+25 =56 wins

M 14+20 =34 N 25+14 =39
U 31+13+12+10=66 wins U 31+20+10 =61 wins

Points	M	V	N	U	
	0	2	1	3	U wins

d) Borda points:

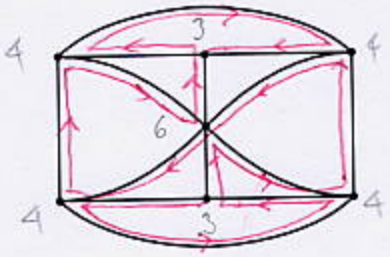
M 14·4+20·3+12·2+54·1 = 194
N 25·4+16·3+39·2+20·1 = 246
U 31·4+35·3+20·2+14·1 = 283 wins
V 30·4+29·3+29·2+12·1 = 277
1000

Uranus wins all contests

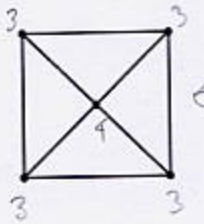
1000 = 100 · 10 ↑ 4+3+2+1, pts per ballot

so it checks out

3. (12pts) Determine whether each of the following graphs has an Euler path or an Euler circuit. If it does, find it, if not, explain why not.

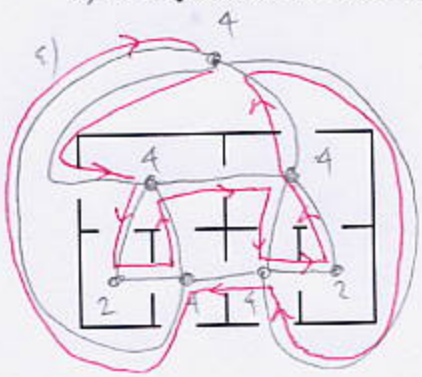


Has exactly two vertices, so has an Euler path, but no Euler circuits



Has more than two odd vertices, so no Euler circuits nor paths

4. (14pts) Below is a floor plan of a section of a museum, with doors joining rooms indicated.
a) Represent the floor plan as a graph (rooms are vertices, don't forget to include an "outside").
b) Use the graph to determine if it is it possible to walk around the section, passing through every door exactly once. If it is, draw the route.
c) Is it possible to do the same as in b), and start and finish outside?



b) Graph has no odd vertices, so has Euler circuits.

c) Yes - a circuit may start at any point of the circuit. (drawn)

5. (20pts) A tourist would like to visit Los Angeles, Las Vegas, Phoenix and San Francisco while trying to minimize the distance traveled. The table below has the distances between the cities.

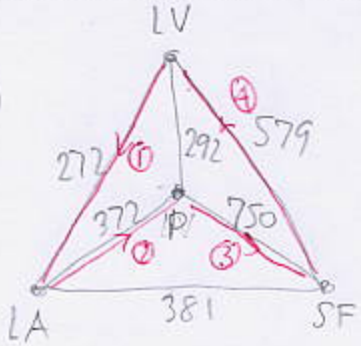
a) Draw a weighted graph that corresponds to this problem.

b) Use the brute force method to find the route that minimizes the distance traveled. First list all the possible orders of visits with Las Vegas as the starting city.

c) Use the nearest neighbor algorithm to find an approximate solution to the problem. Is it the same as in c)?

	LV	LA	P
LA	272		
P	292	372	
SF	579	381	750

a)



b) Brute force:

LV LA P SF LV
272 + 372 + 750 + 579 = 1973

LV LA SF P LV
272 + 381 + 750 + 292 = 1695

LV P LA SF LV
292 + 372 + 381 + 579 = 1624 smallest

LV P SF LA LV
292 + 750 + 381 + 272 = 1695

LV SF LA P LV
579 + 381 + 372 + 292 = 1624

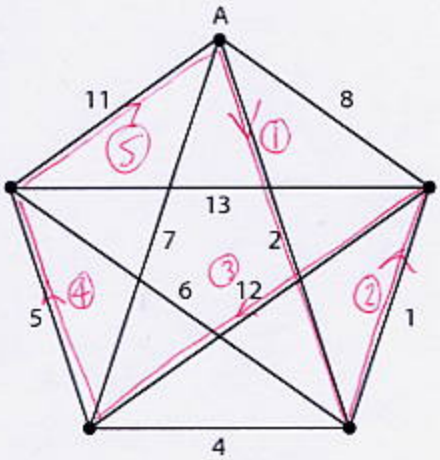
LV SF P LA LV
579 + 750 + 372 + 272 = 1973

c) on graph, length is 1973, worst possible

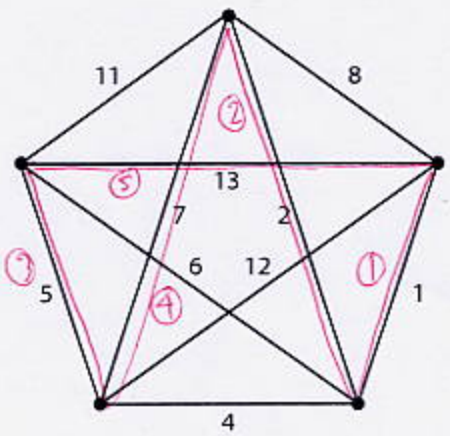
Bonus. (10pts) Find an approximate solution to the traveling salesman problem. Show the weight of the found circuits. Use (one on each picture)

a) the nearest neighbor algorithm starting at A.

b) the greedy algorithm.



2 + 1 + 12 + 5 + 11 = 31



1 + 2 + 5 + 7 + 13 = 28