Mathematical Concepts — Exam 2
MAT 117, Spring 2012 — D. Ivanšić

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Show all your work!

$\frac{a}{b} = \frac{(E)}{1-P(E)}$ $\quad P(E) = \frac{a}{a+b}$ where odds in favor of E are $a : b$ $\quad P(B \mid A) = \frac{n(A \text{ and } B)}{n(A)}$

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$P(A \text{ or } B) = P(A) + P(B)$ (if A and B are mutually exclusive)

$P(A \text{ and } B) = P(A) \cdot P(B \mid A) \quad P(A \text{ and } B) = P(A) \cdot P(B)$ if A and B are independent

$E = P_1 \cdot A_1 + P_2 \cdot A_2 + \cdots + P_n \cdot A_n$

1. (6pts) In a restaurant, there are 17 choices for appetizers, 43 for the main dish and 13 for dessert. Given these choices, how many different three-course meals could you have?

17 43 13 choices

$17 \cdot 43 \cdot 13 = 9503$

2. (6pts) A die is rolled four times. How many different outcomes does this experiment have?

6 6 6 6

$6 \cdot 6 \cdot 6 \cdot 6 = 6^4 = 1296$

3. (14pts) The table shows the pattern of mammography results and breast cancer rates among a number of U.S. women ages 40–50. Assuming the numbers are representative for the general population, what is the probability that a random U.S. woman between the ages of 40 and 50:

a) has breast cancer?
b) had a positive mammogram?
c) does not have breast cancer and had a positive mammogram?
d) has breast cancer, given her mammogram is positive?
e) doesn't have breast cancer, given her mammogram is negative?
f) has a negative mammogram, given that she has breast cancer?

	Cancer	No Cancer	Total
Positive mammo.	720	6,944	7664
Negative mammo.	80	92,256	92,336
Total	800	99,200	100,000

a) $\frac{800}{100,000} = \frac{8}{1000} = 0,008 = \frac{1}{125}$

b) $\frac{7664}{100,000} = 0,07664 = \frac{479}{6250}$

c) $\frac{6944}{100,000} = 0,06944 = \frac{217}{3125}$

d) $\frac{720}{7664} = 0,093946 = \frac{45}{479}$

e) $\frac{92,256}{92,336} = 0,999134 = \frac{3844}{3889}$

f) $\frac{80}{800} = 0,1 = \frac{1}{10}$

4. (18pts) Write the probabilities and odds against and in favor of the following events (you can show any work needed below):

	Event	probability	odds against	odds in favor
a)	Rolling a 4 on a single roll of a die	$\frac{1}{6}$	5:1	1:5
b)	Drawing a red face card from a deck of cards	$\frac{6}{52}=\frac{3}{26}$	46:6 = 23:3	3:23
c)	Getting at least one tail on three coin tosses	$\frac{7}{8}$	1:7	7:1
d)	Getting sum 7 or 8 on a roll of two dice	$\frac{11}{36}$	25:11	11:25
e)	Both numbers odd on a roll of two dice	$\frac{9}{36}=\frac{1}{4}$	27:9 = 3:1	1:3

	happens	doesn't happen
a)	1	5
b)	6	46
c)	7	1
d)	11	25
e)	9	27

12 faces

HHH ✓ THH ✓
HHT ✓ THT ✓
HTH ✓ TTH ✓
HTT ✓ TTT ✓

sum 7 or sum 8 ?

1,6	2,6
2,5	3,5
3,4	4,4
4,3	5,3
5,2	6,2
6,1	

$\underbrace{\qquad}_{11}$

odd odd

$\underline{\quad}$ $\underline{\quad}$

3 choices 3

odd

$3 \cdot 3 = 9$

5. (14pts) A spinner has 8 equal-size fields, one of which is labeled W, two are labeled I and five are labeled N. A game of chance is set up like this: the player pays \$5 and spins. Depending on whether the spinner lands on W, I or N the player wins \$15, \$7 or nothing, respectively.

a) Find the expected value of this game.

b) What is the fair price of this game?

c) If a player played this game 100 times, how much would they expect to win or lose?

outcome	prob.
15-5 = 10	$\frac{1}{8}$
7-5 = 2	$\frac{2}{8}$
-5	$\frac{5}{8}$

a) expected value $= 10 \cdot \frac{1}{8} + 2 \cdot \frac{2}{8} + (-5) \cdot \frac{5}{8} = \frac{10+4-25}{8} = -\frac{11}{8} = -1.375$

(lose \$1.375 per game, on average)

b) fair price $= -1.375 + 5 = 3.625$

c) $100 \cdot 1.375 = 137.50$

expect to lose \$137.50

6. (10pts) In the ice cream bin of a convenience store, 66% of the products contain vanilla, 47% contain chocolate, and 31% have both of those ingredients. If an ice cream product is selected at random, what is the probability that:

a) it contains vanilla or chocolate?

b) it lacks at least one of those ingredients?

a) P(vanilla or choco.) = P(vanilla) + P(choco) − P(vanilla and choco.)

$= 0.66 + 0.47 - 0.31 = 0.82$

b) P(lacks least one of vanilla or choco.) = 1 − P(has both)

$= 1 - 0.31 = 0.69$

7. (14pts) A picky music lover browsing through an online music store finds that, in his opinion, 65% of the tracks there suck. Choosing tracks randomly, what is the probability that he will select

a) on two tries, both tracks that don't suck?

b) on three tries, all three tracks that suck?

c) on four tries, at least one track that doesn't suck?

d. suck = doesn't suck

a) P(1st d. suck and 2nd d. suck) = P(1st d. suck) · P(2nd d. suck)

$= 0.35 \cdot 0.35 = 0.1225$

b) P(1st sucks and 2nd sucks and 3 sucks)

= P(1st sucks) · P(2nd sucks) · P(3rd sucks) $= 0.65 \cdot 0.65 \cdot 0.65 = 0.274625$

c) P(at least one d. suck) = 1 − P(all four suck)

= 1 − P(1st sucks) ... P(4th sucks)

$= 1 - 0.65^4 = 1 - 0.17850\ldots = 0.821494$

8. (18pts) An animal shelter has 7 black kittens, 4 calicos and 5 grey kittens. If you pick two kitties at random, what is the probability that: 16 kittens
a) both are calicos?
b) the first is black and the second is gray?
c) exactly one is grey?

a) P(1st calico and 2nd calico) = P(1st calico)·P(2nd calico | 1st calico)

$= \frac{4}{16}\cdot\frac{3}{15} = \frac{1}{4}\cdot\frac{1}{5} = \frac{1}{20} = 0.05$

b) P(1st black and 2nd grey) = P(1st black)·P(2nd grey | 1st black)

$= \frac{7}{16}\cdot\frac{5}{15} = \frac{7}{16}\cdot\frac{1}{3} = \frac{7}{48} = 0.145833$

c) P(exactly one grey) = P(1st gray and 2nd not gray) + P(1st not gray and 2nd gray)

mutually exclusive

= P(1st gray)·P(2nd not gray | 1st gray) + P(1st not gray)·P(2nd gray | 1st not gray)

$= \frac{5}{16}\cdot\frac{11}{\cancel{15}_{3}} + \frac{11}{16}\cdot\frac{\cancel{5}^{1}}{\cancel{15}_{3}} = \frac{11+11}{48} = \frac{22}{48} = \frac{11}{24} = 0.458333$

Bonus. (10pts) Two cards are drawn from a deck at random. What is the probability that the first one is a face card and the second one is a club? *Hint: this will help you somewhere in the problem: if B and C are mutually exclusive, $P(B \text{ or } C \mid A) = P(B \mid A) + P(C \mid A)$.*

Actually, hint is misleading. Do it another way:

P(1st face and 2nd club) = P(1st club face and 2nd club face OR
1st club face and 2nd club number OR
1st non-club face and 2nd club face OR
1st non-club face and 2nd club number)

mutually exclusive

= P(1st club face and 2nd club face) + ... 3 more

= P(1st club face)·P(2nd club face | 1st club face) + P(1st club face)·P(2nd club number | 1st club face)

+ P(1st non-club face)·P(2nd club face | 1st non-club face) + P(1st non-club face)·P(2nd club number | 1st non-club face)

$= \frac{3}{52}\cdot\frac{2}{51} + \frac{3}{52}\cdot\frac{10}{51} + \frac{9}{52}\cdot\frac{3}{51} + \frac{9}{52}\cdot\frac{10}{51} = \frac{6+30+27+90}{52\cdot 51} = \frac{\cancel{153}}{\cancel{52\cdot 51}} = \frac{3}{52}$

$= 0.0576923$