Mathematical Concepts — Exam 1
MAT 117, Spring 2012 — D. Ivanšić

Name: Saul Ocean

Show all your work!

$I = Prt$ $A = P(1+rt)$ $A = P\left(1+\frac{r}{n}\right)^{nt}$ $A = P\frac{\left(1+\frac{r}{n}\right)^{nt}-1}{\frac{r}{n}}$ $P = PMT\frac{1-\left(1+\frac{r}{n}\right)^{-nt}}{\frac{r}{n}}$ $Y = \left(1+\frac{r}{n}\right)^{n}-1$

1. (10pts) a) 12 is what percent of 25?

$A = PB$

$12 = P \cdot 25$

$P = \frac{12}{25} = 0.48$

48%

b) 7 is 64% of what?

$7 = 0.64 \cdot B$

$B = \frac{7}{0.64} = 10.9375$

2. (10pts) You borrowed \$400 from a friend, and repaid him in 5 months with \$450. What simple annual interest rate did you pay on this loan?

$A = P(1+rt)$

$450 = 400\left(1+r\cdot\frac{5}{12}\right) \quad |\div 400$

$1.125 = 1+r\cdot\frac{5}{12} \quad |-1$

$0.125 = r\cdot\frac{5}{12} \quad |\cdot\frac{12}{5}$

$0.3 = r$

3. (8pts) How much should you deposit in an account bearing 3.47%, compounded semi-annually, if you would like to have \$4,000 in three years?

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$4000 = P\left(1+\frac{0.0347}{2}\right)^{2\cdot 3}$

$4000 = P\cdot 1.1087.. \quad |\div 1.1087..$

$P = \frac{4000}{1.1087..} = 3607.76013$

Round up to

\$3607.77

4. (14pts) In 2010, single man Fidelio filed income taxes His total income was \$85,300, he deposited \$5000 into a retirement account, paid \$7400 in mortgage interest, \$2100 in property taxes, \$3900 in state income taxes and donated \$750 to charity. Use the table below to first determine Fidelio's taxable income (don't forget the exemption) and then find the tax on this income.

Income bracket	Tax rate
up to \$8,375	10%
\$8,376–\$34,000	15%
\$34,001–\$82,400	25%
\$82,401–\$171,850	28%
\$171,851–\$373,650	33%
more than \$373,650	35%
exemption	\$3650

Total subtractions:

$5000 + 7400 + 2100 + 3900 + 750 + 3650$

$= 22,800$

Taxable income $= 85,300 - 22,800$

$= 62,500$

Tax on 62,500 is $0.10 \cdot 8375 + 0.15(34000 - 8375) + 0.25(62,500 - 34000)$

$= 837.50 + 3843.15 + 7125$

$= 11,806.25 \leftarrow$ tax

5. (14pts) You would like to save up for a nice new computer.
a) How much should you deposit every day into an account with 3.5% interest, compounded daily, in order to have \$2,500 in three years?
b) How much of the final amount is from deposits and how much from interest?

a) $A = P\dfrac{\left(1+\frac{r}{n}\right)^{nt}-1}{\frac{r}{n}}$

$2500 = P\dfrac{\left(1+\frac{0.035}{365}\right)^{365\cdot 3}-1}{\frac{0.035}{365}}$ (1095)

$2500 = P \cdot 1154.4952\ldots \quad | \div 1154.49\ldots$

$P = \dfrac{2500}{1154.45\ldots} = 2.16544\ldots$

About \$2.17 per day.

b) Total deposits:

$365 \cdot 3 \cdot 2.17 = 2376.15$

Interest: $2500 - 2376.15$

$= 123.85$

6. (32pts) True story: in an attempt at reducing Demi's anger at his indiscretions, Ashton bought her a very nice car for \$103,000. Made-up part: for this he took on a 7-year loan at 5.25%, compounded monthly.

a) What is his montly payment on the loan?

b) What are his total payments over the course of the loan? How much of this amount is for interest?

c) How much of his first payment goes toward interest, and how much towards the principal?

d) How much does he owe after 3 years?

a) $P = PMT \dfrac{1-\left(1+\frac{r}{n}\right)^{-nt}}{\frac{r}{n}}$ $\quad \overset{84}{\underset{12\cdot 7}{}}$

$103{,}000 = PMT \dfrac{1-\left(1+\frac{0.0525}{12}\right)^{-12\cdot 7}}{\frac{0.0525}{12}}$

$103{,}000 = PMT \cdot 70.167\ldots \quad | \div 70.167$

$PMT = \dfrac{103{,}000}{70.167} = 1467.92$

b) Total Payments

$= 12\cdot 7 \cdot 1467.92 = 123{,}305.28$

Total interest $= 123{,}305.28 - 103{,}000$

$= 20{,}305.28$

c) $I = Prt$

$= 103{,}000 \cdot 0.0525 \cdot \frac{1}{12}$

$= 450.63 \leftarrow$ towards interest

$1467.92 - 450.63 = 1017.29$ ✓

towards principal

d) recalculate payments with $-12\cdot 4$

$P = 1467.92 \dfrac{1-\left(1+\frac{0.0525}{12}\right)^{-12\cdot 4}}{\frac{0.0525}{12}}$

$= 1467.92 \cdot 43.210\ldots$

$= 63429.04$

7. (12pts) If you deposit \$2000 into an account bearing 11% interest, compounded quarterly, how long will it take until you have \$6000 in the account?

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$6000 = 2000\left(1+\frac{0.11}{4}\right)^{4\cdot t} \quad | \div 2000$

$3 = (1.0275)^{4t} \quad | \log$

$\log 3 = \log(1.0275)^{4t}$

$\log 3 = 4t \log(1.0275) \quad | \div 4\log(1.0275)$

$t = \frac{\log 3}{4\log(1.0275)} = 10.12 \text{ years}$

Bonus. (10pts) In an effort to save for a down payment on a home, Maria deposits \$3000 into an account, bearing 4.5%, compounded monthly. Her financial situation does not allow her to make any additional deposits for a year. Then, after getting a promotion at work, she starts making monthly deposits of \$200 for the next two years into the same account. How much is in the account in three years from now?

Think of this as two accounts:

1) single deposit of 3000 for 3 years $\longrightarrow$: $A = P\left(1+\frac{r}{n}\right)^{nt} = 3000\left(1+\frac{0.045}{12}\right)^{12\cdot 3}$

$= 3000 \cdot 1.144\ldots$

$= 3432.74$

2) a series of deposits of 300 for two years

$A = P\,\frac{\left(1+\frac{r}{n}\right)^{nt}-1}{\frac{r}{n}}$

$= 200 \cdot \frac{\left(1+\frac{0.045}{12}\right)^{12\cdot 2}-1}{\frac{0.045}{12}}$

$= 200 \cdot 25.06\ldots$

$= 5012.81$

Total in both "accounts":

$3432.74 + 5012.81 = 8445.55$