College Algebra — Joysheet 10 **MAT 140, Fall 2012 — D. Ivanšić**	**Name:** Saul Ocean *Show all your work!*

1. (5pts) If $\log_a 4 = 0.489301$ and $\log_a 5 = 0.568061$, find (show how you obtained your numbers):

$\log_a 20 = \log_a (4 \cdot 5)$
$= \log_a 4 + \log_a 5$
$= 0.489301 + 0.568061$
$= 1.057362$

$\log_a \frac{16}{5} = \log_a \frac{4^2}{5} = \log_a 4^2 - \log_a 5$
$= 2 \log_a 4 - \log_a 5$
$= 2 \cdot 0.489301 - 0.568061 = 0.410541$

2. (11pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_3 (81x^4y^2) = \log_3 81 + \log_3 x^4 + \log_3 y^2 = 4 + 4\log_3 x + 2\log_3 y$

$\log_5 \sqrt[5]{\frac{x^3y^{-7}}{25y^3}} = \log_5 \left(\frac{x^3y^{-7}}{25y^3}\right)^{\frac{1}{5}} = \frac{1}{5}\log_5\left(\frac{x^3}{25y^{10}}\right) = \frac{1}{5}\left(\log_5 x^3 - \log_5 25 - \log_5 y^{10}\right)$

$= \frac{1}{5}\left(3\log_5 x - 2 - 10\log_5 y\right)$

3. (12pts) Write as a single logarithm. Simplify if possible.

$\frac{1}{3}\ln(125x^{\frac{1}{2}}) + \frac{1}{2}\ln(625y^7) - \ln(x^{\frac{2}{3}}) = \ln (125x^{\frac{1}{2}})^{\frac{1}{3}} + \ln (625y^7)^{\frac{1}{2}} - \ln x^{\frac{2}{3}}$

$= \ln\left(\frac{(5x^{\frac{1}{6}})(25y^{\frac{7}{2}})}{x^{2/3}}\right) = \ln\left(125x^{-\frac{1}{2}}y^{\frac{7}{2}}\right) = \ln\left(\frac{125y^{\frac{7}{2}}}{x^{1/2}}\right)$

$2\log_a(x+6) - 4\log_a(\underbrace{x^2-36}_{(x+6)(x-6)}) - 3\log_a(x-6) = \log_a (x+6)^2 - \log_a\left((x+6)(x-6)\right)^4 - \log_a (x-6)^3$

$= \log_a \frac{(x+6)^{\cancel{2}\ 0}}{(x+6)^{\cancel{4}}_{2}(x-6)^4(x-6)^3} = \log_a \frac{1}{(x+6)^2(x-6)^7} = -\log_a\left((x+6)^2(x-6)^7\right)$

Solve the equations.

4. (5pts) $7^{2-x} = 49^{3x-1}$

$$7^{2-x} = (7^2)^{3x-1}$$
$$7^{2-x} = 7^{6x-2}$$
$$2-x = 6x-2 \quad | +x+2$$
$$4 = 7x$$
$$x = \frac{4}{7}$$

5. (7pts) $4^{x+9} = 9^{2x+4}$ $\quad | \ln$

$$\ln 4^{x+9} = \ln 9^{2x+4}$$
$$(x+9)\ln 4 = (2x+4)\ln 9$$
$$x\ln 4 + 9\ln 4 = 2x\ln 9 + 4\ln 9$$
$$x\ln 4 - 2x\ln 9 = 4\ln 9 - 9\ln 4$$
$$x(\ln 4 - 2\ln 9) = 4\ln 9 - 9\ln 4$$
$$x = \frac{4\ln 9 - 9\ln 4}{\ln 4 - 2\ln 9} = 1.225918$$

6. (8pts) $\log_2(x+1) + \log_2(x+5) = 5$

$$\log_2((x+1)(x+5)) = 5 \quad | 2^{-}$$
$$2^{\log_2((x+1)(x+5))} = 32$$
$$(x+1)(x+5) = 32$$
$$x^2+6x+5 = 32$$
$$x^2+6x-27 = 0$$
$$(x+9)(x-3) = 0$$
$$x = -9, 3$$

Check: $\log_2(-8) + \log_2(-4) = 5$ not defined

$\log_2(4) + \log_2(8) = 5$ ✓

$2 + 3 = 5$

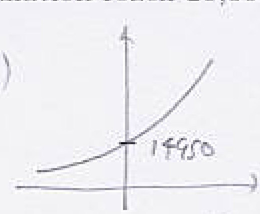
$\boxed{\text{Only } x=3 \text{ is the solution}}$

7. (12pts) The 2000 and 2010 censuses recorded Murray as having 14,950 and 17,741 people, respectively. Assume Murray's population grows exponentially.
a) Write the function describing the number $P(t)$ of people t years after 2000. Then find the exponential growth rate of Murray's population.
b) Graph the function.
c) According to this model, when will the population reach 20,000?

a) $P(t) = 14,950\, e^{kt}$

$$14950\, e^{k\cdot 10} = 17,741$$
$$e^{10k} = \frac{17741}{14950} \quad | \ln$$
$$\ln e^{10k} = \ln\left(\frac{17741}{14950}\right)$$
$$10k =$$
$$k = \frac{\ln\left(\frac{17741}{14950}\right)}{10} \approx 0.0171167$$

b)



c) $14950\, e^{kt} = 20000$ (solve for t)

$$e^{kt} = \frac{20000}{14950} \quad | \ln$$
$$kt = \ln\frac{20000}{14950}$$
$$t = \frac{\ln\left(\frac{20000}{14950}\right)}{k} \approx 17.002161$$

About 17 years since 2000, so in 2017.