

Simplify, so that the answer is in form $a + bi$.

1. (4pts) $(7 - i)(4 + 5i) = 28 + 35i - 4i - 5i^2 = 33 + 31i$

2. (6pts) $\frac{2 + 3i}{1 - 8i} = \frac{2 + 3i}{1 - 8i} \cdot \frac{1 + 8i}{1 + 8i} = \frac{2 + 16i + 3i + 24i^2}{1^2 - (8i)^2} = \frac{-22 + 19i}{65} = -\frac{22}{65} + \frac{19}{65}i$

3. (4pts) Simplify and justify your answer.

$i^{131} = i^{128} \cdot i^3 = (i^4)^{32} \cdot i^3 = \underbrace{i \cdot i \cdot i \cdot i}_{=1} \cdot i^3 = \underbrace{i \cdot i \cdot i}_{-1} = -i$

Solve the equations:

4. (6pts) $4x^2 + 4x = 3x + 14 \quad | -3x - 14$

$4x^2 + x - 14 = 0$

$x = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 4 \cdot (-14)}}{2 \cdot 4} = \frac{-1 \pm \sqrt{225}}{8}$

$= \frac{-1 \pm 15}{8} = -\frac{16}{8}, \frac{14}{8} = -2, \frac{7}{4}$

6. (6pts) Solve by completing the square.

$x^2 - 12x = 8 \quad | + 6^2$

$x^2 - 2 \cdot 6x + 6^2 = 8 + 6^2$

$(x + 6)^2 = 44$

$x + 6 = \pm \sqrt{44}$

$x = -6 \pm 2\sqrt{11}$

5. (8pts) $3x^4 - 22x^2 - 45 = 0$

Set $u = x^2$

$3u^2 - 22u - 45 = 0$

$u = \frac{22 \pm \sqrt{(-22)^2 - 4 \cdot 3 \cdot (-45)}}{2 \cdot 3}$

$= \frac{22 \pm \sqrt{484 + 540}}{6} = \frac{22 \pm \sqrt{1024}}{6}$

$= \frac{22 \pm 32}{6} = \frac{54}{6}, -\frac{10}{6} = 9, -\frac{5}{3} = u$

$x^2 = 9 \quad x^2 = -\frac{5}{3}$

$x = \pm 3 \quad x = \pm \sqrt{\frac{5}{3}}i$

7. (12pts) The quadratic function $f(x) = x^2 - 6x + 13$ is given. Do the following without using the calculator.

- Find the x -intercepts of its graph, if any. Find the y -intercept.
- Find the vertex of the graph.
- Sketch the graph of the function.

a) $x^2 - 6x + 13 = 0$

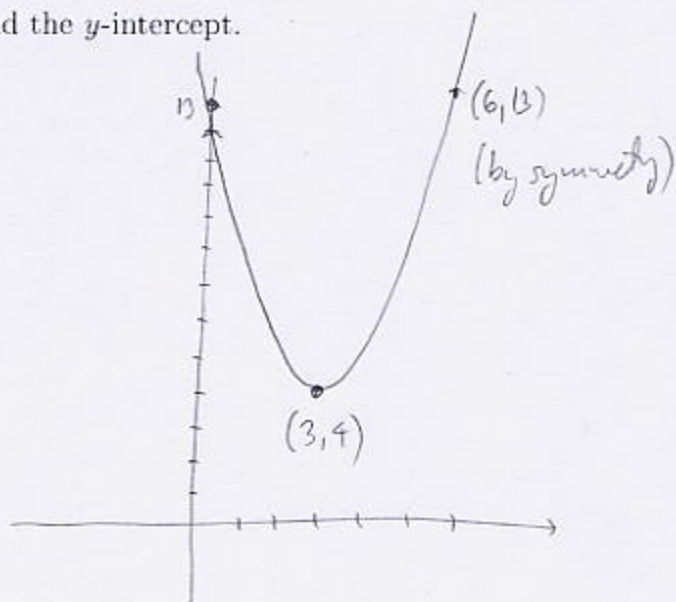
$$x = \frac{6 \pm \sqrt{(-6)^2 - 4 \cdot 13}}{2} = \frac{6 \pm \sqrt{-16}}{2} \quad \text{no real solutions}$$

x -int - none

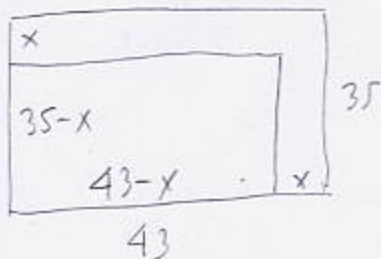
y -int: $f(0) = 13$

b) vertex = $-\frac{b}{2a} = -\frac{-6}{2 \cdot 1} = 3$

$f(3) = 3^2 - 6 \cdot 3 + 13 = 4$



8. (14pts) Ella mows a rectangular yard of size 43×35 yards. She finds that her 1505 square yards are too much work to mow, so she decides to decrease the size of the mowing area by reducing the length and width by the same amount. If her target yard area is 900 square yards, by how much should she reduce the length and width of her current yard?



x = amount that length, width are reduced

$$(35-x)(43-x) = 900$$

$$1505 - 43x - 35x + x^2 = 900 \quad | -900$$

$$x^2 - 78x + 605 = 0$$

$$x = \frac{78 \pm \sqrt{(-78)^2 - 4 \cdot 1 \cdot 605}}{2 \cdot 1} = \frac{78 \pm \sqrt{3664}}{2} = \frac{78 \pm 4\sqrt{229}}{2}$$

$$= 39 \pm 2\sqrt{229} = 69.265492, \boxed{8.734508}$$

Since 69.26 is too big to be subtracted from 35, 43, 8.734508 is the only solution.