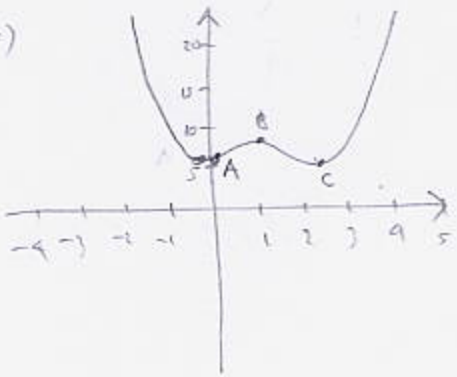
College Algebra — Joysheet 5
MAT 140, Fall 2012 — D. Ivanšić

Name: Saul Ocean

Show all your work!

1. (10pts) Use your calculator to accurately sketch the graph of the function $f(x) = x^4 - 5x^3 + 7x^2 - 2x + 6$. Draw the graph here, and indicate units on the axes.
a) Find the local maxima and minima for this function.
b) State the intervals where the function is increasing and where it is decreasing.

a)



A = (0.17368, 5.838508)
B = (1.223668, 7.114927)
C = (2.352652, 5.566097)

b) Increasing on $(0.17368, 1.223668) \cup (2.352652, \infty)$
Decreasing on $(-\infty, 0.17368) \cup (1.223668, 2.352652)$

2. (20pts) Let $f(x) = \sqrt{x-3}$, $g(x) = \dfrac{2x+1}{3x-16}$. Find the following (simplify where possible):

$(f+g)(2) = f(2) + g(2)$
$= \sqrt{2-3} + \frac{5}{-10}$ not defined
not def.

$(fg)(7) = f(7) \cdot g(7) = \sqrt{7-3} \cdot \frac{15}{5} = 2 \cdot 3 = 6$

$\frac{f}{g}(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{x-3}}{\frac{2x+1}{3x-16}} = \frac{\sqrt{x-3}}{1} \cdot \frac{3x-16}{2x+1}$
$= \frac{(3x-16)\sqrt{x-3}}{2x+1}$

$(f \circ g)(7) = f(g(7)) = f\left(\frac{15}{5}\right) = f(3) = \sqrt{3-3} = 0$

$(g \circ f)(x) = g(f(x)) = g(\sqrt{x-3}) = \dfrac{2\sqrt{x-3}+1}{3\sqrt{x-3}-16}$

State the domain of $(fg)(x)$ = intersection of domains of f, g

Domain f: must have $x - 3 \geq 0$
$x \geq 3$

Domain of g: can't have $3x - 16 = 0$
$3x = 16$
$x = \frac{16}{3}$

Domain of fg: $[3, \frac{16}{3}) \cup (\frac{16}{3}, \infty)$

3. (8pts) Consider the function $h(x) = \sqrt{3+2x}$. Find functions f and g so that $h(x) = f(g(x))$. Find two different solutions to this problem, neither of which is the "stupid" one.

$h(x) = \sqrt{3+2x}$

$g(x) = 3+2x$ $\quad f(x) = \sqrt{x}$

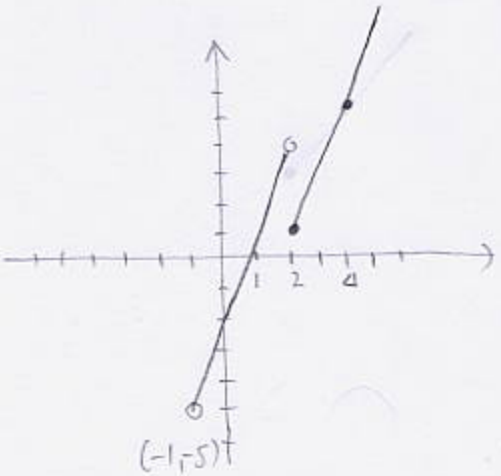
$g(x) = 2x$ $\quad f(x) = \sqrt{3+x}$

4. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} 3x-2, & \text{if } -1 < x < 2 \\ 2x-3, & \text{if } x \geq 2. \end{cases}$$

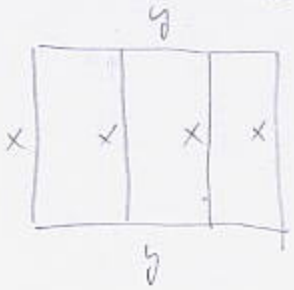
x	$3x-2$
-1	-5
2	4

x	$2x-3$
2	1
4	5



5. (14pts) Farmer Charles has 1500 meters of fencing. He would like to enclose a rectangular plot of land and divide it in three parts with fences parallel to one side of the rectangle.
a) Express the area of the enclosure as a function of the length of one of the sides x. What is the domain of this function?
b) Graph the function in order to find the maximum. What are the dimensions of the enclosure that has the greatest area?

a)



$4x+2y = 1500 \quad |-4x$

$2y = 1500-4x \quad |\div 2$

$y = 750-2x$

$A = xy = x(750-2x)$

$= -2x^2+750x$

Domain: $x \geq 0$ and $y \geq 0$

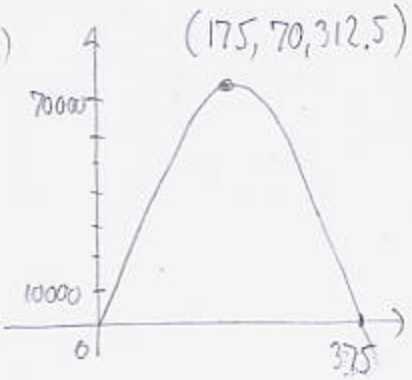
$750-2x \geq 0 \quad |+2x$

$750 \geq 2x \quad |\div 2$

$x \leq 375$

Domain $= [0, 375]$

b)



$x = 187.5$

$y = 375$

Area $= 70312.5 \text{ m}^2$