Mathematical Concepts — Joysheet 2
MAT 117, Fall 2012 — D. Ivanšić

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Show all your work!

Solve the equations.

1. (3pts) $9+2(1-x)=7+2(x-1)$

$9+2-2x=7+2x-2$

$11-2x=5+2x \quad |+2x$

$11-4x=5 \quad |-11$

$-4x=-6 \quad |\div -4$

$x=\frac{-6}{-4}=\frac{3}{2}$

2. (6pts) $x^2-2x=x+28 \quad |-x-28$

$x^2-3x-28=0$

$(x-7)(x+4)=28$

$x-7=0$ or $x+4=0$

$x=7$ or $x=-4$

3. (5pts) Solve the equation for r:

$A=P(1+rt) \quad |\div P$

$\frac{A}{P}=1+rt \quad |-1$

$\frac{A}{P}-1=rt \quad |\div t$

$r=\frac{\frac{A}{P}-1}{t}\cdot\frac{P}{P}$

$r=\frac{A-P}{Pt}$

Simplify.

4. (8pts) $\frac{3x+7}{2x^2+9x-5}-\frac{2x+1}{x^2+x-20}=\frac{3x+7}{(2x-1)(x+5)}-\frac{2x+1}{(x+5)(x-4)}$

factors of 27: prod = 27 | ±27, ±1 | ±9, ±3 | ±13 | no

prod = −10, sum = 9: 10, −1

$2x^2+10x-x-5$

$=2x(x+5)-(x+5)$

$=(2x-1)(x+5)$

$=\frac{(3x+7)(x-4)-(2x+1)(2x-1)}{(2x-1)(x+5)(x-4)}$

$=\frac{3x^2-5x-28-(4x^2-1)}{(2x-1)(x+5)(x-4)}=\frac{-x^2-5x-27}{(2x-1)(x+5)(x-4)}=-\frac{x^2+5x+27}{(2x-1)(x+5)(x-4)}$

doesn't factor

5. (8pts) $\frac{3+\frac{4x+7}{x^2-9}}{x-\frac{10}{x+3}}=\frac{\frac{3(x^2-9)+4x+7}{x^2-9}}{\frac{x(x+3)-10}{x+3}}=\frac{\frac{3x^2+4x-20}{x^2-9}}{\frac{x^2+3x-10}{x+3}}$

prod = −60, sum = 4: −10, 6

$3x^2+10x+6x-20$

$=x(3x+10)-2(3x+10)$

$=(3x+10)(x-2)$

$=\frac{(3x+10)(x-2)}{(x-3)(x+5)}\cdot\frac{x+3}{(x+5)(x-2)}=\frac{3x+10}{(x-3)(x+5)}$

Simplify, showing intermediate steps.

6. (2pts) $\sqrt{96} =$ $\sqrt{6\cdot 16} = \sqrt{6}\cdot 4 = \boxed{4\sqrt{6}}$

7. (4pts) $\sqrt{216x^3y^4} =$ $\sqrt{6\cdot 36\cdot x^2\cdot x\cdot (y^2)^2} = \sqrt{6}\cdot 6\cdot |x|\sqrt{x}\,|y^2| = \boxed{6|x|y^2\sqrt{6x}}$

always positive

8. (5pts) $\sqrt[3]{50x^5y}\sqrt[3]{10x^2y^2} =$ $\sqrt[3]{500x^7y^3} = \sqrt[3]{4\cdot 125\cdot x^6\cdot x\cdot y^3} = \sqrt[3]{4}\cdot 5\cdot x^2\cdot \sqrt[3]{x}\cdot y$

$(x^2)^3$

$= \boxed{5x^2y\sqrt[3]{4x}}$

9. (8pts) Simplify. Express answers first in terms of positive exponents, then convert to root notation.

$\left(32x^{\frac{1}{2}}y^{\frac{5}{4}}\right)^{\frac{2}{5}} 7x^{\frac{4}{15}}\left(8y^{\frac{7}{2}}\right)^{-\frac{5}{3}} =$ $32^{\frac{2}{5}}\, x^{\frac{1}{2}\cdot\frac{2}{5}}\, y^{\frac{5}{4}\cdot\frac{2}{5}}\, 7x^{\frac{4}{15}}\, 8^{-\frac{5}{3}}\, y^{\frac{7}{2}\cdot(-\frac{5}{3})}$

$= \left(\sqrt[5]{32}\right)^2 x^{\frac{1}{5}} y^{\frac{1}{2}}\cdot 7x^{\frac{4}{15}}\cdot\frac{1}{(\sqrt[3]{8})^5}\, y^{-\frac{35}{6}} = \frac{4\cdot 7}{32} x^{\frac{1}{5}+\frac{4}{15}} y^{\frac{1}{2}-\frac{35}{6}} = \frac{7}{8} x^{\frac{7}{15}} y^{-\frac{32}{6}}$

$= \frac{7x^{\frac{7}{15}}}{8y^{16/3}} = \boxed{\frac{7\sqrt[15]{x^7}}{8\sqrt[3]{y^{16}}}}$

10. (5pts) Simplify.

$(\sqrt{7}+3\sqrt{2})(\sqrt{2}-5\sqrt{7}) =$

$= \sqrt{14} + 3\sqrt{2}^2 - 5\sqrt{7}^2 - 15\sqrt{14} = 3\cdot 2 - 5\cdot 7 + \sqrt{14} - 15\sqrt{14}$

$6 \qquad 35$

$= -29 - 14\sqrt{14}$

11. (6pts) Rationalize the denominator.

$\frac{\sqrt{7}-1}{2+\sqrt{6}}\cdot\frac{2-\sqrt{6}}{2-\sqrt{6}} = \frac{2\sqrt{7}-2-\sqrt{42}+\sqrt{6}}{2^2-\sqrt{6}^2} = \frac{2\sqrt{7}-2-\sqrt{42}+\sqrt{6}}{-2} = \frac{2+\sqrt{42}-2\sqrt{7}-\sqrt{6}}{2}$

$4-6=-2$